MIXING ANGLES, B-MESON LIFE-TIME IN THE SIX QUARK MODEL BASED ON $SO(10) \times U(1)_{PQ}$

V.P. NAIR

Institute for Advanced Studies, Princeton, NJ, USA

and

Louis MICHEL and Kameshwar C. WALI*

IHES, Bures-sur-Yvette, France

We show that a recently proposed multigenerational, grand unified model with three parameters (top-quark mass and two phases of Higgs vacuum expectation values) yields results in good agreement with the most recent phenomenological limits placed on the weak mixing angles of the quarks. These limits take into account the experimental value for B-meson life-time.

Institut des Hautes Etudes Scientifiques
35, route de Chartres
91440 - Bures-sur-Yvette (France)

Decembre 1983

IHES/P/83/77

* Permanent Address : Physics Dept., Syracuse University, Syracuse, NY 13210, USA.

The recent determination [1] of B-meson life-time has provided an important piece of experimental information which enables one to put stringent restrictions on the Kobayashi-Maskawa mixing angles in the six-quark scheme. Revised phenomenological fits [2-4] show that the bottom-charm (b-c) quark transition matrix element compared with up-strange (u-s) transition is very small, implying that the weak mixing angles θ_2 and θ_3 are quite small. This poses a serious challenge to theoretical models, because the implied hierarchy seems to be in an apparent contradiction with the expectations for the values of the mixing angles in terms of quark mass ratios.

The purpose of this note is to discuss in some detail the consequences of a recently proposed model [5] on this question. Three generations of quarks and leptons are considered within the framework of a grand unified theory based on the group SO(10) combined with the global, axial, U(1) Peccei-Quinn [6] symmetry $(U(1)_{PQ})$. The Peccei-Quinn symmetry plays a dual role in the model; (i) it eliminates the strong CP-Violation problem : (ii) it acts as horizontal flavor symmetry, and distinguishes the different generations. We shall confine ourselves only to a few relevant features of the model here. The interested reader should consult reference 5 for further details.

With each generation of fermions belonging to a $\underline{16}$-dimensional spinorial representation of SO(10) , and with the choice of $\underline{10}$ (complexified) and $\underline{126}$ representations for the Higgs scalars that couple to the fermions, the quark mass matrices are complex, symmetric matrices with the generic form M ,

$$M = \begin{bmatrix} 0 & Ae^{i\alpha} & 0 \\ Ae^{i\alpha} & 0 & Be^{i\beta} \\ 0 & Be^{i\beta} & Ce^{i\gamma} \end{bmatrix} \qquad (1)$$

which can be written as

$$M = PXP \ , \tag{2}$$

where

$$P = \mathrm{diag}(e^{i(\alpha-\beta+\gamma/2)}\ , e^{i(\beta-\gamma/2)}\ , e^{i\gamma/2})\ ,$$

and

$$X = \begin{bmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{bmatrix} . \tag{3}$$

Thus X is a real symmetric matrix with every element being positive. A real orthogonal matrix O diagonalizes X ,

$$OXO^T = \mathrm{diag}(m_1, -m_2, m_3)\ , \tag{4}$$

where $0 < m_1 < m_2 < m_3$. They are the values of the current quark masses. <u>The most important feature</u> [7] <u>of the model is that both</u> X <u>and</u> O <u>can be expressed in terms of</u> m_1, m_2, m_3 .

$$A = \left[\frac{m_1 m_2 m_3}{m_1 - m_2 + m_3}\right]^{1/2}\ , \quad B = \left[\frac{(m_3+m_1)(m_3-m_2)(m_2-m_1)}{m_1-m_2+m_3}\right]^{1/2}, \tag{5}$$

$$C = m_3 - m_2 + m_1$$

$$O = \begin{bmatrix} \left[\frac{m_2 m_3 (m_3-m_2)}{(m_3-m_1)(m_2+m_1)(m_3-m_2+m_1)}\right]^{1/2} & \left[\frac{m_1(m_3-m_2)}{(m_3-m_1)(m_2+m_1)}\right]^{1/2} & -\left[\frac{m_1(m_2-m_1)(m_3+m_1)}{(m_3-m_1)(m_2+m_1)(m_3-m_2+m_1)}\right]^{1/2} \\ +\left[\frac{m_1 m_3 (m_3+m_1)}{(m_3+m_2)(m_2+m_1)(m_3-m_2+m_1)}\right]^{1/2} & -\left[\frac{m_2(m_3+m_1)}{(m_3+m_2)(m_2+m_1)}\right]^{1/2} & +\left[\frac{m_2(m_3-m_2)(m_2-m_1)}{(m_3+m_2)(m_2+m_1)(m_3-m_2+m_1)}\right]^{1/2} \\ \left[\frac{m_1 m_2 (m_2-m_1)}{(m_3+m_2)(m_3-m_1)(m_3-m_2+m_1)}\right]^{1/2} & \left[\frac{m_3(m_2-m_1)}{(m_3+m_2)(m_3-m_1)}\right]^{1/2} & \left[\frac{m_3(m_3-m_2)(m_3+m_1)}{(m_3-m_2+m_1)(m_3+m_2)(m_3-m_1)}\right]^{1/2} \end{bmatrix}$$

Then, as in reference 5, let $X^{(d)}$ and $X^{(u)}$ correspond to the mass matrices $M^{(d)}$ and $M^{(u)}$ in the down- and up-charge sectors,

$$M^{(d)} = P^{(d)} X^{(d)} P^{(d)} \ , \ M^{(u)} = P^{(u)} X^{(u)} P^{(u)} \ , \tag{7}$$

where $P^{(d)}$ and $P^{(u)}$ are the corresponding diagonal, pure phase matrices. If $O^{(d)}$ and $O^{(u)}$ are the desired orthogonal matrices that diagonalise $X^{(d)}$ and $X^{(u)}$,

$$O^{(d)} X^{(d)} O^{(d)T} = \mathrm{diag}(m_d, -m_s, m_b) \tag{8}$$

$$O^{(u)} X^{(u)} O^{(u)T} = \mathrm{diag}(m_u, -m_c, m_t) \ , \tag{9}$$

where $m_d, \ldots, m_t$ denote the masses of the indicated quarks, then the Cabibbo-Kobayashi-Maskawa matrix in the charged current interaction, namely,

$$\mathcal{U}_c = U_L^{(u)} U_L^{(d)\dagger} \tag{10}$$

is given by

$$\mathcal{U}_c = Q[O^{(u)} P^{(u)*} P^{(d)} O^{(d)T}]R \ , \tag{11}$$

with the matrix in the rectangular parantheses determined completely by the parameters that enter in the mass matrix (vacuum expectation values, coupling constants) and Q and R being two arbitrary diagonal pure phase matrices. They reflect the arbitrary phases of the quark fields. We define

$$P = P^{(u)*}P^{(d)} = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}) \; ,$$

$$c_{ij} = \sum_k O^{(u)}_{ik} O^{(d)}_{jk} \cos \phi_k \; , \tag{12}$$

$$s_{ij} = \sum_k O^{(u)}_{ik} O^{(d)}_{jk} \sin \phi_k \; ,$$

and choose Q and R so that U_c reduces to the standard Kobayashi-Maskawa form. Then the required mixing angles $\theta_1, \theta_2, \theta_3$ and $\mathrm{Im}(U_c)_{12}$ that is related to the weak CP-violation parameter are given by

$$\cos \theta_1 = (U_c)_{11} = \sqrt{c_{11}^2 + s_{11}^2} \; , \tag{13}$$

$$-\sin \theta_1 \cos \theta_2 = (U_c)_{21} = -\sqrt{c_{21}^2 + s_{21}^2} \tag{14}$$

$$\sin \theta_1 \cos \theta_3 = (U_c)_{12} = \sqrt{c_{12}^2 + s_{12}^2} \tag{15}$$

$$-\sin \theta_2 \sin \theta_3 \sin \delta = \mathrm{Im}(U_c)_{22}$$

$$= \frac{(s_{11}c_{22}+c_{11}s_{22})(c_{12}c_{21}-s_{12}s_{21})-(c_{11}c_{22}-s_{11}s_{22})(s_{12}c_{21}+c_{12}s_{21})}{\cos \theta_1 \cos \theta_2 \cos \theta_3 \sin^2 \theta_1} \tag{16}$$

From (12) - (16), it follows that the mixing angles and CP-violation depend on the six quark masses and two phase angle differences. As only the topquark mass is unknown, the model contains only three unknown parameters - the top quark mass m_t and two phase differences, say

$$\alpha = (\phi_1-\phi_2) \; , \; \beta = (\phi_2-\phi_3) \quad (\text{then} \;\; \phi_1-\phi_3 = \alpha+\beta) \; . \tag{17}$$

The mixing angles and $\mathrm{Im}(U_c)_{22}$ are given by

$$\cos^2\theta_1 = K_o+K_1\cos\alpha+K_2\cos(\alpha+\beta)+K_3\cos\beta \; , \tag{18}$$

$$\sin^2\theta_1\cos^2\theta_2 = L_o+L_1\cos\alpha+L_2\cos(\alpha+\beta)+L_3\cos\beta \; , \tag{19}$$

$$\sin^2\theta_1\cos^2\theta_3 = N_o+N_1\cos\alpha+N_2\cos(\alpha+\beta)+N_3\cos\beta \; , \tag{20}$$

$$\begin{aligned} &\cos\theta_1\cos\theta_2\cos\theta_3\sin^2\theta_1\mathrm{Im}(U_c)_{22} \\ &= A_1\sin\alpha+A_2\sin(\alpha+\beta)+A_3\sin\beta \\ &+ \sin\alpha\,(A_4\cos(\alpha+\beta)+A_5\cos\beta) \\ &+ \sin(\alpha+\beta)(A_6\cos\alpha+A_7\cos\beta) \\ &+ \sin\beta\,(A_8\cos\alpha+A_9\cos(\alpha+\beta)) \; , \end{aligned} \tag{21}$$

where K_i, L_i, N_i, and A_i are all given functions of masses. We take the standard typical values for the quark masses, namely,

$$\begin{aligned} &m_d = 7.5 \text{ MeV} \qquad m_s = 150 \text{ MeV} \qquad m_b = 5000 \text{ MeV} \\ &m_u = 5 \text{ MeV} \qquad m_c = 1250 \text{ MeV} \qquad m_t > 30 \text{ GeV} \; , \end{aligned}$$

and compute these functions for varying values of m_t. The results for $K_o \ldots N_3$ are given in Table 1. The results for A's are similar. Notice that they are very slowly varying smooth functions of m_t . Having these numbers at our disposal, we study the choice of other parameters.

The most stringent requirement arises from the very well determined Cabibbo angle [2],

$$\sin\theta_1 = 0.231 \pm 0.003 .$$

First of all, it rules out the choice $\alpha = \beta = 0$, which would have implied that $\mathrm{Im}(U_c)_{22} = 0$. In other words, there would have been no weak CP-violation in the conventional way due to gauge bosons in the charged interactions. One then had to appeal to the Higgs sector for CP-violation. Secondly, the study of the numbers in Table 1 shows that K_1 contributes most dominantly, K_2 and K_3 being relatively of no significance to the value of θ_1 . The value $\alpha = 90°$ leads to

$$\sin\theta_1 = 0.2264$$

for $m_t = 30 - 100$ GeV . This is remarkably close to the lower limit $\sin\theta_1 = 0.228$ set by experiments. The variation of $\sin\theta_1$ from 0.228-0.234 allows the variation of α from 91.5° to 97° . We take $\alpha = 94°$ to yield

$$\sin\theta_1 = 0.231 \tag{22}$$

independent of m_t when it is varied from 30 GeV to 100 GeV . Having fixed α this way, we vary β to set limits on its variation for various values of m_t . For this purpose, we assume the limits provided in reference [2],

$$\begin{aligned} 0.015 < \sin\theta_2 < .09 , \\ \sin\theta_3 < .04 , \end{aligned} \tag{23}$$

which include the recent experiments [1],

$$\left|(U_c)_{bc}\right| = 0.053 \begin{matrix} + .010 \\ - .009 \end{matrix} \tag{24}$$

on B-meson life-time in their analysis. The results are plotted in Fig.1 and Fig.2. They show that for $m_t = 30 - 100$ GeV, we can find β such that all the experimental constraints are satisfied quite well.

Finally, we come to the CP-violation effect predicted by the model. The $K^0-\overline{K}^0$ transition matrix M_{12} from the standard relevant box graph [4] is given by

$$M_{12} = -\frac{G_F^2 M_W^2}{16\,\pi^2}\left(\sum_{i,j=u,c,t} \lambda_i \lambda_j A_{ij}\right) M_{12,vac}\, B\,, \tag{25}$$

where

$$M_{12,vac} = \frac{4}{3}\, f_K^2\, m_K \tag{26}$$

with $f_K \simeq 1.23\, m_\pi$, is the vacuum insertion contribution and B is a constant characterizing the deviation of the vacuum-insertion calculation from unity [8]. The other quantities appearing in (25) are defined in reference [4]. We calculate the quantity

$$M = \Sigma \lambda_i \lambda_j A_{ij} \tag{27}$$

for $m_t = 30$-100 GeV. The results for $\mathrm{Im}\, M_{12}$ [9] are plotted in Fig. 3. Inserting the value of $M_{12,vac}$, we find

$$\mathrm{Im}\, M_{12} = -(.114 \times 10^{-13} \mathrm{MeV}) \times cB\, \mathrm{Im}\, M \quad , \tag{28}$$

where $c = 1.0223 \times 10^7$. In order that we do not conflict with experiments, $cB \, \mathrm{Im} \, M \leq 1$.

The main points of our results can be read from the figures. For convenience, we summarize them in Table 2. The most stringent limits on β are provided by Eq. 24. We note that the values $m_t > 90$ GeV are excluded, both from the constraint of Eq. 24 and $cB \, \mathrm{Im} \, M \leq 1$, if we take $B \simeq \frac{1}{3}$. For each value of m_t , an allowed range of β emerges from Fig.1. In this range, $\sin \theta_2$, $\sin \theta_3$, $\sin \delta$, and $c \, \mathrm{Im} \, M$ are slowly varying, increasing functions of β . We have given in Table 2 the values of the above mentioned quantities for the end points of the allowed range of β . It is worth noting that $(U_c)_{bc}$ involves a specific combination of sines and cosines of all the mixing angles and the Kobayashi-Maskawa phase δ . By restricting the absolute value of this matrix element, the model predicts, for all the investigated values of m_t , limits for $\sin \theta_2$ and $\sin \theta_3$,

$$.038 \leq \sin \theta_2 \leq .057 \; , \quad .011 \leq \sin \theta_3 \leq .022 \; .$$

These limits are more stringent than the current phenomenological constraints. The model also predicts $\sin \delta$ to lie between $.909 \leq \sin \delta \leq .999$. Thus, for each value of the top-quark mass, the complete Kobayashi-Maskawa matrix is known within certain limits. Consequently, the model provides a rich body of results that can be compared with experiments, once the value of m_t is known.

In conclusion, the generic form for the mass matrix in (1) seems to provide a good and satisfactory description of low energy parameters including the new piece of information concerning B-meson life-time. Such a form for the mass matrix was suggested a long time ago by Fritzsch [10] from heuristic considerations. Here it is derived within the framework of a grand unified theory combined with Peccei-Quinn symmetry which eliminates the strong CP-violation problem. The axion can be made to be phantom axion [11]. Thus the model is a realistic one and the results obtained show that it merits a serious study. We have used a minimal model consisting of only 10 and 126 Higgs representations that couple to the fermions. Addition of 120 representation introduces an antisymmetric component in the mass matrix; Stech [12] has recently analysed such a general situation. The results in the two cases appear, at least qualitatively, the same. The presently established experimental results do not warrant the addition of an antisymmetric component.

We note that the results are very sensitive to numerical approximations. The assumed quark masses and hence the hierarchies in the quark mass ratios suggest linear expansions in terms of the mass ratios which have often been used in the literature evaluating the Kobayashi-Maskawa matrix elements. However, comparison of such expansions with exact numerical evaluations used in this paper shows that there are serious discrepancies. There are delicate cancellations in the first order expansions leading to significant contributions from higher orders in mass ratios. In such models, therefore it seems advisable to do careful numerical work without resorting to approximation. We also note that we have deliberated avoided a very careful study of the weak CP-violation including both the real and imaginary parts of M_{21} . Our reason for this is that the model has a rich Higgs structure, and there is bound to be CP-violation due to Higgs exchanges. A full understanding of the latter mechanism requires a detailed study of the Higgs potential of the model, which includes in addition to the representations 10 and 126, those that are necessary to break the SO(10) symmetry down to that of the Lie algebra $SU(3) \times SU(2) \times U(1)$. This and other implications of the model (rare decay modes, charm and B-meson physics) are currently under study.

Footnotes and References.

[1] N.S. Lockyer et al., Phys. Rev. Lett. 50, 1542 (1983).

[2] K.Kleinknecht and B. Renk, Preprint, CERN-EP/83-126 (submitted to Physics Letters). References to their earlier work are contained in this paper. See also ref. 3.

[3] P.H. Ginspor, S.L. Glashow, M.B. Wise, Phys. Rev. Lett. 50, 1415 (1983).

[4] For a comprehensive review of both the theoretical and phenomenological background and references, see Ling-Lie Chau, Phys. Reports, 95, 1-14 (1983). Earlier limits on the mixing angles are given in this report.

[5] A. Davidson, V.P. Nair, and K.C. Wali, Syracuse University preprints SU-4217-257, SU-4222-265, 1983. Submitted for publication in Phys. Rev.

[6] R. Peccei and H. Quinn, Phys. Rev. Lett. 38, 1440 (1979).

[7] H. Georgi and D.V. Nanopolous, Nucl. Phys. B 109 (1979) 51. These authors discuss a model based on O(10) and arrive at the generic forms in (1) for the quark mass matrices. They carry out the diagonalization in the Appendix of the paper.

[8] J.F. Donohue , E. Golowich, and B.R. Holstein, Phys. Lett. 119 B, (1982) 412. $B = 0.33$ in their evaluation. This is regarded as the most model independent evaluation of the constant. For earlier evaluations of B based on bag model, see ref. 4.

[9] We only consider $\mathrm{Im}\, M_{12}$, because the real part is quite sensitive to low energy contributions. See D. Hochberg and R.G. Sachs, Phys. Rev. D.27, 606 (1983). See also L. Bergström et al, ref. TH.3659-CERN (1983). As stated later in our paper, our model does not warrant a detailed calculation of CP-violation effect, until we analyze the Higgs sector.

[10] H. Fritzsch, Phys. Lett. 73B, (1978) 317, Nucl. Phys. B155 (1979) 189.

[11] M. Dine, W. Fishler, and M. Srednicki, Phys. Lett. 104B (1981) 199. M.B. Wise, H. Georgi, and S.L. Glashow, Phys. Rev. Lett. 47, 402,(1981).

[12] B. Stech, Phys. Lett. 130B (1983) 189.

<u>TABLE 1</u>

m_t in GeV	K_0	K_1	K_2	K_3	L_0	L_1	L_2	L_3	N_0	N_1	N_2	N_3
30	0·9487	0·0259	9·578 E-4	1·307 E-5	0·0481	-0·0259	-9·193 E-4	3·136 E-3	0·0510	-0·0259	-9·285 E-4	2·534 E-4
40	0·9487	0·0260	8·250 E-4	1·132 E-5	0·0485	-0·0261	-7·999 E-4	2·744 E-3	0·0510	-0·0261	-7·997 E-4	2·195 E-4
50	0·9487	0·0261	7·355 E-4	1·012 E-5	0.0488	-0·0261	-7·175 E-4	2·469 E-3	0·0511	-0·0261	-7·130 E-4	1·963 E-4
60	0·9487	0·0262	6·700 E-4	9·242 E-6	0·0490	-0·0262	-6·563 E-4	2·263 E-3	0·0511	-0·0262	-6·495 E-4	1·792 E-4
70	0·9487	0·0262	6·193 E-4	8·556 E-6	0·0491	-0·0262	-6·084 E-4	2·101 E-3	0·0511	-0·0262	-6·004 E-4	1·659 E-4
80	0·9487	0·0262	5·787 E-4	8·003 E-6	0·0492	-0·0262	-5·697 E-4	1·970 E-3	0·0511	-0·0263	-5·610 E-4	1·552 E-4
90	0·9487	0·0263	5.451 E-4	7·546 E-6	0·0493	-0·0263	-5·376 E-4	1·860 E-3	0·0511	-0·0263	-5·284 E-4	1·463 E-4
100	0·9487	0·0263	5·168 E-4	7·158 E-6	0·0493	-0·0263	-5·104 E-4	1·767 E-3	0·0511	-0·0263	-5·010 E-4	1·388 B-4

Dependence of the coefficients $K_o,\ldots,K_3$, $L_0,\ldots,L_3$, $N_0,\ldots,N_3$ on the top-quark mass. See Equs. 18-20 for the definitions.

TABLE 2

m_t in GeV	β	$\sin\theta_2$	$\sin\theta_3$	$c\ \mathrm{Im}\ M$	$\sin\delta$
30	9°-16°	·0397-·0567	·0177-·0225	1·013-2·101	·9994-·9994
40	13°-20°	·0379-·0572	·0160-·0214	1·019-2·552	·9862-·9927
50	13°-20°	·0387-·0560	·0148-·0200	1·055-2·797	·9123-9852
60	10°-19°	·0382-·0563	·0132-·0192	1·064-3·233	·9360-·9790
70	3°-17	·0385-·0566	·0109-·0185	0·9731-3·666	·9092-·9740
80	0°-14°	·0457-·0568	·0120-·0177	1·686-4·055	·9383-·9698
90	0°-9°	·0523-·0564	·0138-·166	2·961-4·259	·9563-·9653
100	-	-	-	-	-

By restricting $|(U_c)_{bc}|$, we find allowed ranges for β , $\sin\theta_2$, $\sin\theta_3$, $\sin\delta$, and $c\ \mathrm{Im}\ M$.

Figure Captions.

Fig.1 : $|(U_c)_{bc}|$ as a function of β in degrees for various values of m_t in GeV . The dotted horizontal lines represent the phenomenological constraints : Eq. 24.

Fig.2 : $\sin\theta_2$ as a function of β in degrees for various values of m_t in GeV. The dotted horizontal lines represent the phenomenological constraints. $\sin\theta_3$ is always less than .04 . We do not plot it, but give its values in Table 2.

Fig.3 : $c\ \mathrm{Im}\ M$ as a function of β in degrees for various values of m_t in GeV . The dotted horizontal line represents the constraint $cB\ \mathrm{Im}\ M \leq 1$, where $B = 1/3$.

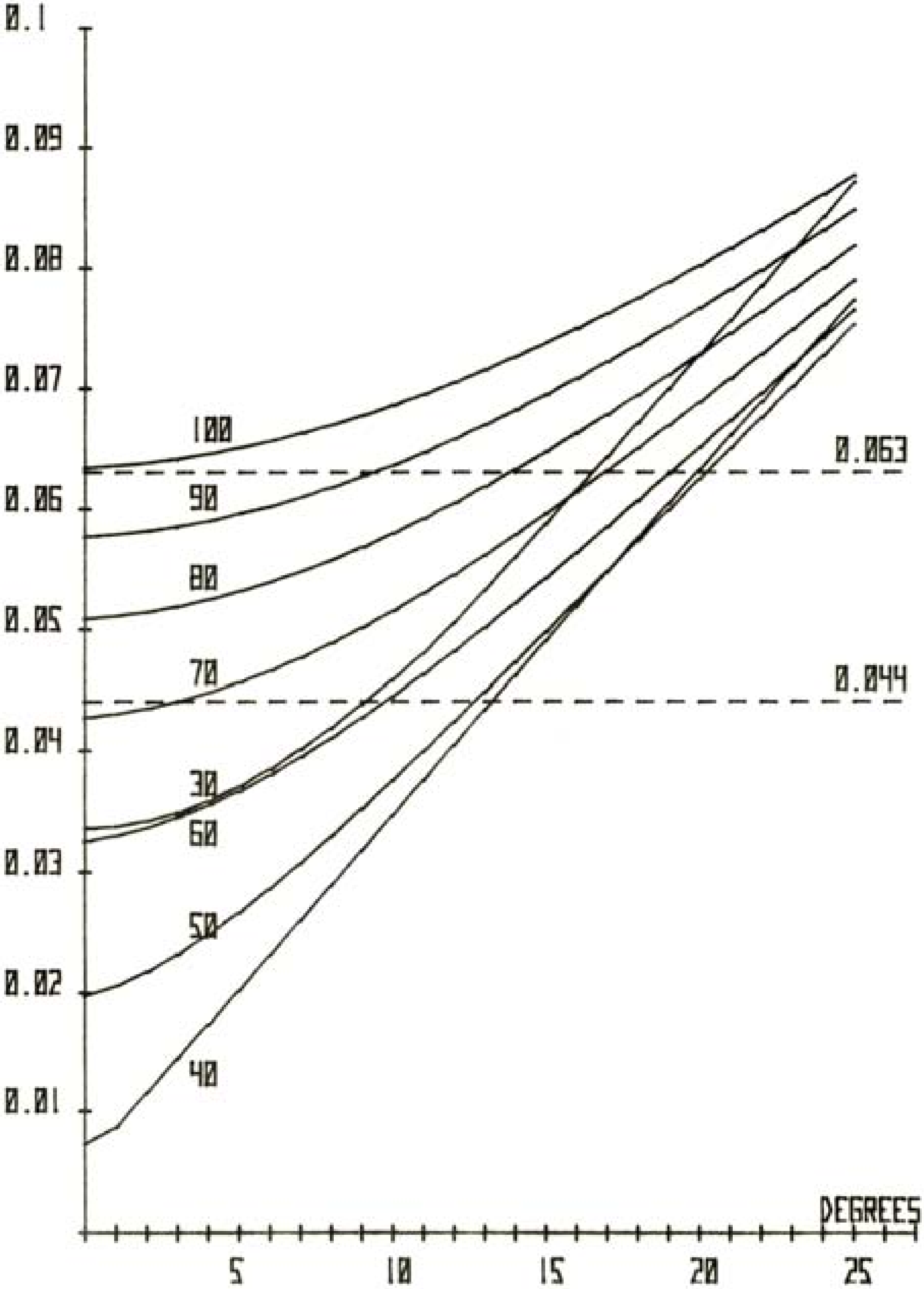
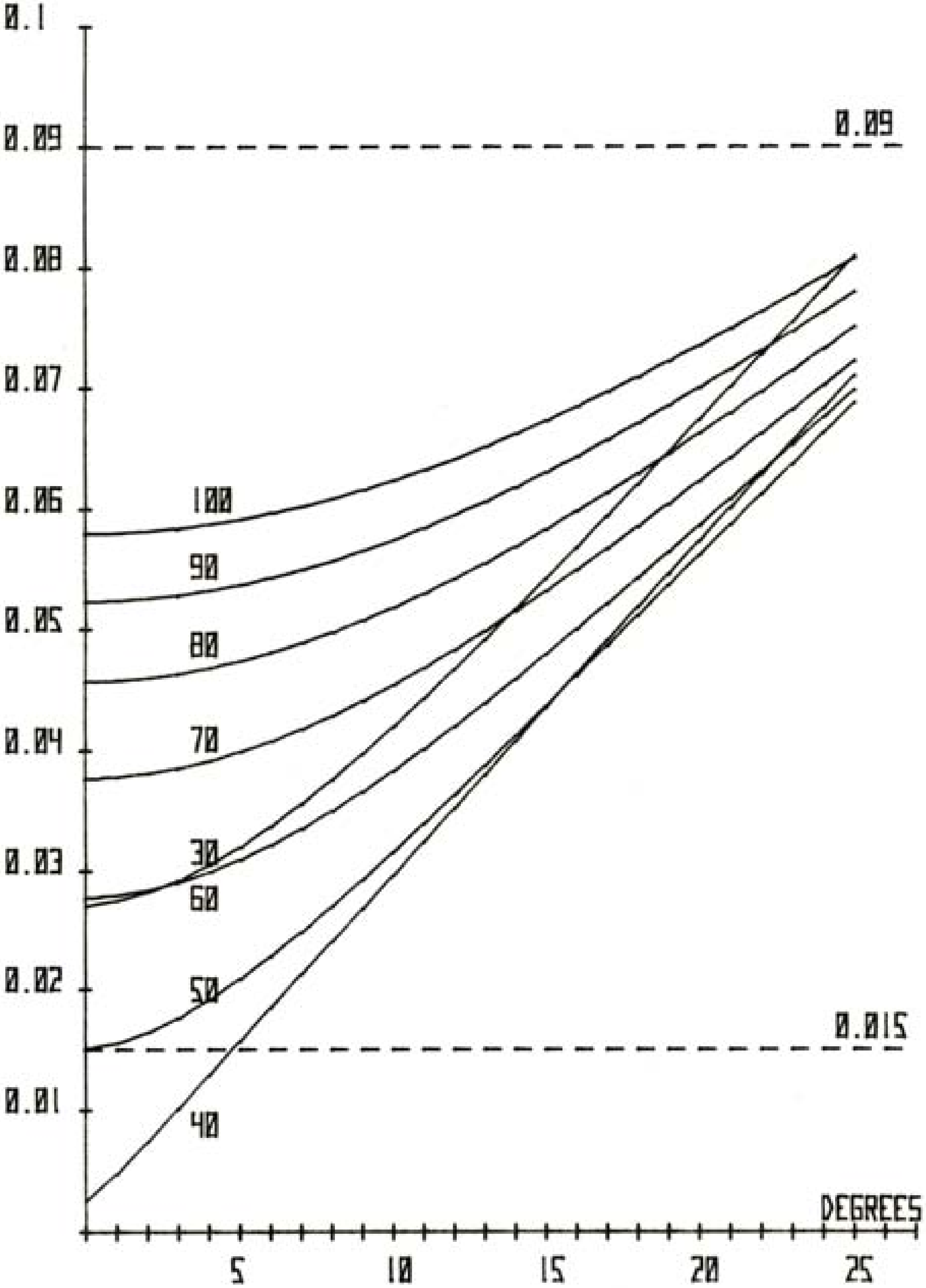

0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
100
90
80
70
30
60
50
40
0.063
0.044
DEGREES
5
10
15
20
25

FIGURE 1



0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.09
0.015
100
90
80
70
30
60
50
40
DEGREES
5
10
15
20
25

FIGURE 2

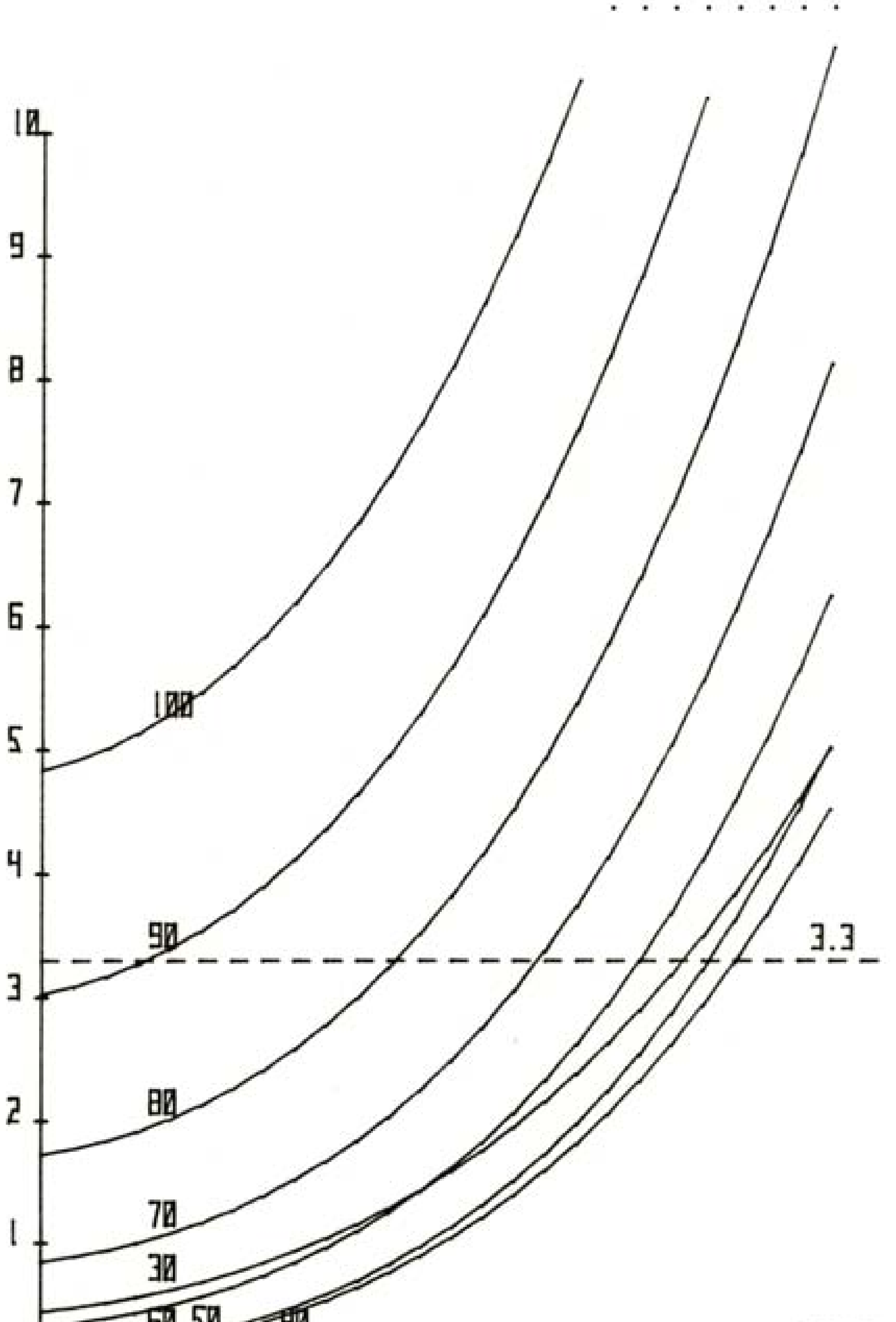


FIGURE 3.