MONOPOLE FIELDS FROM VORTEX SHEETS RECONCILING ABELIAN AND CENTER DOMINANCE

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We describe a new order parameter for the confinement-deconfinement transition in lattice SU(2) Yang-Mills theory. It is expressed in terms of magnetic monopole field correlators represented as sums over sheets of center vortices. Our construction establishes a link between "abelian" and "center dominance". It avoids an inconsistency in the treatment of small scales present in earlier definitions of monopole fields by respecting Dirac's quantization condition for magnetic fluxes.

1. Abelian and center dominance

It is widely believed that confinement in SU(2) Yang-Mills theory is due to condensation of topological defects, but it is still debated whether the relevant defects are center vortices or magnetic monopoles.

In the first scenario [1] confinement is usually discussed in terms of area decay for the Wilson loop. In lattice theory, the location of vortex sheets giving rise to area law have been identified with surfaces of plaquettes, p, where $sign[TrU_{\partial p}] = -1$ (thin vortices), [2]. Here U_μ denotes the $SU(2)$-gauge field, and $U_{\partial p}$ the Wilson plaquette, [more precisely, if p is described by a lattice site x and two direction μ, ν then $U_{\partial p} \equiv U_{\mu\nu}(x) \equiv U_\mu(x)U_\nu(x+\hat{\nu})U^\dagger(x+\hat{\nu})U^\dagger_\nu(x)$]. However it was later proved [3] that close to the continuum, at zero temperature, thin vortices form a dilute gas and hence they are unable to induce an area decay of the Wilson loop. Therefore, to explain confinement, one needs to invoke either "thick vortices" [4], where $sign[TrU_L] = -1$ for loops L comprising more than four links, or, presumably equivalently [5], P-vortices which are defined as follows [6]: one introduces a gauge fixing (maximal center gauge) by maximizing $\sum_{x,\mu}(TrU_\mu(x))^2$. In this gauge one defines the $\mathbf{Z}_2$-gauge field $Z_\mu(x) = sign[TrU_\mu(x)]$; the location of P-vortices is identified with the set of plaquettes where $Z_{\partial p} = Z_{\mu\nu}(x) = -1$. We refer to the above scenario, where the center degrees of freedom are believed to be the relevant ones for confinement, as "center dominance".

't Hooft put on a concrete basis a proposal explaining confinement as a condensation of magnetic monopoles [7] : he suggested [8] to construct a scalar field $X(U)$ with values in $su(2)$, as a function of the gauge field U and transforming in the adjoint representation of the gauge group $SU(2)$. By requiring that $X(U)$ be diagonal he then fixes a gauge ("abelian projection"). The resulting theory exhibits a residual $U(1)$ gauge invariance.

The argument of the diagonal component of the $SU(2)$ gauge field in this "abelian projection gauge" plays the role of a compact $U(1)$ "photon" field, A_μ, with range $(-2\pi, 2\pi)$, and the off-diagonal components are

described by a complex field, c, charged with respect to the residual $U(1)$ gauge group. The points in space-time where the two eigenvalues of the matrix X coincide identify the positions of the monopoles in this gauge. Confinement is believed to emerge as a consequence of monopole–condensation in the form of a "dual Meissner effect".

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We refer to this scenario for confinement (together with the assumption that off-diagonal degrees of freedom are irrelevant for a description of low energy physics) as "abelian dominance".

2. U(1) monopole order parameter

In [9], a monopole field operator $\hat{m}$ is proposed which plays the role of an order parameter for the confinement–deconfinement

transition in the "abelian dominance" scenario, i.e., with vanishing v.e.v. in the deconfinement phase and v.e.v. $\langle \hat{m} \rangle \neq 0$ in the confinement phase.

A euclidean representation of the two-point monopole correlation function $G(x,x') = \langle \hat{m}(x)\hat{m}^\dagger(x') \rangle$ is

constructed as follows: let $E^x_\mu, x = (x^0, \vec{x})$ denote the (lattice) electric Coulomb field generated on the 3–dimensional sublattice, $\mathbf{Z}^3_{x^0}$, at constant euclidean time $x^0 \in \mathbf{Z}$, by a unit charge located at $\vec{x} \in \mathbf{Z}^3$. Denoting by Δ_{x^0} the

3–dimensional (lattice) laplacian in $\mathbf{Z}^3_{x^0}$, one has:

$$E^x_i(y) = \partial_i \Delta_3^{-1}(y,x), i = 1,2,3 \ , \ E^x_0(y) = 0, \ (1)$$

for $y \in \mathbf{Z}^4$ with $y^0 = x^0$, and $E^x_\mu(y) = 0$ elsewhere, so that $\sum_\mu \partial_\mu E^x_\mu(y) = \delta_x(y)$. Let $B^x_{\nu\rho\sigma}$ denote the * dual of E^x_μ, supported on cubes in the dual lattice, let $j^{xx'}_\mu$ be a unit current supported on a path from x to x' [$\sum_\mu \partial_\mu j^{xx'}_\mu(y) = \delta_x(y) - \delta_{x'}(y)$], and let $\omega^{xx'}_{\nu\rho\sigma}$ denote its * dual, supported on cubes dual to the links in the support of $j^{xx'}_\mu$.

The Yang-Mills action is defined by

$$S(U) = -\beta \sum_p Tr U_{\partial p} = -\beta \sum_{y,\mu,\nu} Tr U_{\mu\nu}(y), \quad (2)$$

and we define a modified action $S_{\omega^{xx'}}(U|X, B^x, B^{x'})$, depending on a unit-norm $su(2)$ scalar X determining an abelian projection, by multiplying $U_{\mu\nu}(y)$ in the plaquette term in (2) by

$$e^{iX(y)2\pi \sum_{z,\rho} \partial_\rho \Delta^{-1}(y,z)[\omega^{xx'}_{\mu\nu\rho} + B^x_{\mu\nu\rho} - B^{x'}_{\mu\nu\rho}](z)} \quad (3)$$

where Δ is the 4-d lattice laplacian.

The 2-point monopole correlation function corresponding to X proposed in [9] can be defined as the Yang-Mills v.e.v. of the disorder field

$$D_{\omega^{xx'}}(B^x, B^{x'}) = e^{-[S_{\omega^{xx'}}(U|X,B^x,B^{x'}) - S(U)]}. \quad (4)$$

Since X transforms under the adjoint representation, the v.e.v. of D is $SU(2)$–gauge invariant.

To make definition (4) plausible, we notice that, in an abelian projection gauge, and after integrating out the "charged field" c, the $SU(2)$ theory appears as a $U(1)$-gauge theory. In this $U(1)$ theory, the disorder field (4) is constructed [10] by translating the field strenghth of $A_\mu(y)$ by $\pi \sum_{z,\rho} \partial_\rho \Delta^{-1}(y,z)[\omega^{xx'} + B^x - B^{x'}]_{\mu\nu\rho}(z)$ in the action. Wegner-'t Hooft duality [11] maps a pure $U(1)$ gauge theory onto

a non–compact abelian Higgs (n.c.H.) model, exchanging the role of monopoles and charges. One can prove [13] that for a $U(1)$ version of the disorder field (4), one has the duality:

$$\langle D_{\omega^{xx'}}(B^x, B^{x'}) \rangle_{U(1)} =$$
$$\langle e^{i(\theta(x) - \theta(x'))} e^{i \sum_{y,\mu} (E^x_\mu - E^{x'}_\mu)(y) A_\mu(y)} \rangle_{n.c.H.} \quad (5)$$

where θ is the charged field and A_μ the gauge field of the dual Higgs model. The r.h.s. of (5) appears as the two-point correlation function of the non-local gauge-invariant charged field of the Higgs model constructed according to Dirac's ansatz [12]. If we expand the l.h.s. [r.h.s.] of (5) in terms of worldlines of monopoles [charges], such worldlines have a source at x and a sink at x'. The $B[E]$ current distributions emerging from these points describe a cloud of soft "photons". Notice that choosing $\omega^{xx'}_{\mu\nu\rho} = \omega^x_{\mu\nu\rho} - \omega^{x'}_{\mu\nu\rho}$, with ω^x dual to a path at constant time x^0 from x to ∞ (where suitable b.c. are imposed so that still $\partial_\mu (*\omega^{xx'})_\mu = \delta_x - \delta_{x'}$) one recognizes the sum $B^x_{\mu\nu\rho} + \omega^x_{\mu\nu\rho}$ as the lattice magnetic field at euclidean time x^0 of a monopole located at $\vec{x}$, with $\omega^x_{\mu\nu\rho}$ playing the role of its Dirac string. It has been rigorously proved in [13] that as $|x - x'| \to \infty$, the correlation function (5) tends to 0 in the deconfined phase of the U(1) gauge theory [Coulomb phase of the dual Higgs model] and approaches a finite value in the confined phase [superconducting Higgs phase]. Hence the monopole

field operator reconstructed from the v.e.v. of the disorder field correlation (5) in U(1) theory is a good order parameter for the confinement-deconfinement transition. On the basis of these arguments it has been claimed in [9] that the monopole field operator reconstructed from v.e.v. of the disorder field (4) is a good order parameter in SU(2) Yang-Mills theory. Numerical evidence in favour of this conjecture emerged in [14]; (see also [10] [15]). Critical exponents associated with this transition extracted from the behaviour of the v.e.v. of (4) appear to be independent of the choice of X [16].

3. Inconsistency and cure

In spite of its great numerical success, the order parameter based on (4) is inconsistent in the treatment of small scales, because it violates Dirac's quantization of fluxes required for self-consistency of a theory where dynamical charges (in our case represented by c) and monopoles coexist. This inconsistency shows up in an unphysical dependence on the "Dirac string" $\omega^{xx'}$ exhibited by $\langle D_{\omega^{xx'}}(B^x, B^{x'})\rangle$. In the abelian projection gauge-fixed theory, this feature appears because the U(1)-gauge theory obtained by integrating out c has a dual which is a compact Higgs model, with dynamical charges and monopoles. The 2-point correlation function of the charged field constructed according to Dirac's ansatz then depends on the choice of the Dirac surfaces swept out by Dirac strings attached monopole worldlines. Let us explain how this happens [17]. In the compact dual Higgs model, the Dirac surfaces, S, are described by integer-valued surface currents, $n_{\mu\nu}$, supported on the plaquettes dual to S. A change of Dirac surfaces, $S \to S'$, for a fixed configuration of monopole worldlines, corresponds to the shift

$$n_{\mu\rho} \to n_{\mu\rho} + \partial_\mu V_\rho - \partial_\rho V_\mu, \tag{6}$$

where V_μ is the integer current supported on the dual of the cubes contained in the volume whose boundary is the closed surface $S' - S$. In the partition function, the interaction of the electric currents generated by the charged particles, whose worldlines are described by an integer 1-current j_μ, with the Dirac surfaces of the monopoles is of the form

$$ieg \sum_{y,z,\mu,\rho} j_\mu(y)\partial_\rho \Delta^{-1}(y,z) n_{\rho\mu}(z) \tag{7}$$

where e is the electric charge of the matter field and g the magnetic charge of the monopole field. The change (6) induces a shift of (7) by

$$ieg \sum_{y,\mu} j_\mu(y) V_\mu(y) \tag{8}$$

which when exponentiated is unity, as required, provided it is an integer multiple of $2\pi i$ [Dirac quantization condition for fluxes]. This happens in the partition function if Dirac's quantization condition for charges holds, i.e. $eg = 2\pi q, q$ an integer, because j_μ and V_μ are integer currents. In the Dirac ansatz for the 2-point function of the charged field, however, j_μ acquires additional Coulomb-like terms, E_μ, which are real-valued, hence

$$eg \sum_{y,\mu} E_\mu(y) V_\mu(y) \notin 2\pi\mathbf{Z} \tag{9}$$

even if $eg \in 2\pi\mathbf{Z}$, and the Dirac strings of monopoles become unphysically "visible". An obvious cure for this inconsistency would be to replace the Coulomb field E^x_μ by a "Mandelstam string" j^x_μ [18], squeezing the entire flux of E^x into a single line from x to ∞ at fixed time (and adding suitable b.c.).

However, this squeezing of the flux is so strong that it produces IR divergences [$(\sum_{y,z,\mu}(E^x_\mu - E^{x'}_\mu)(y)\Delta^{-1}(y-z)(E^x_\mu - E^{x'}_\mu)(z) < \infty$ but $\sum_{y,z,\mu}(j^x_\mu - j^{x'}_\mu)(y)\Delta^{-1}(y-z)(j^x_\mu - j^{x'}_\mu)(z) = \infty$].

To avoid these divergences, we need to replace a fixed Mandelstam string by a sum over fluctuating Mandelstam strings weighted by a measure $\mathcal{D}\nu_q(j^x_\mu)$ such that, in the scaling limit,

$$\int \mathcal{D}\nu_q(j^x_\mu) \quad e^{ie\sum_{y,\mu} j^x_\mu(y) A_\mu(y)} \sim e^{ie\sum_{y,\mu} E^x_\mu(y) A_\mu(y)} \tag{10}$$

[The integer q in the measure $\mathcal{D}\nu_q$ is the one appearing in the Dirac quantization condition

$eg = 2\pi q$]. It has been shown in [17] that a measure with such properties can be constructed as follows: Consider a 3-dimensional XY model supported on a lattice at constant time x^0, with the $U(1)$ spin field, χ, of period $2\pi q$ minimally coupled, with charge e, to the compact gauge field A_μ of the compact Higgs model. Denote by $\langle\cdot\rangle^{x^0}(A)$ the corresponding expectation value, with a coupling constant of the XY model chosen sufficiently large that the symmetry $\chi \to \chi + const$ is spontaneously broken. The correlation functions of the field χ can be expressed in terms of $\mathbf{Z}/q-$ valued currents; in particular

$$\left(\langle e^{i\chi(x)} e^{-i\chi(\infty)}\rangle^{x^0}(A)\right)_{ren} \sim \int \mathcal{D}\nu_q(j_\mu^x) e^{ie\sum_{y\mu} j_\mu^x(y) A_\mu(y)} \tag{11}$$

where $(\cdot)_{ren}$ involves a multiplicative renormalization taking care of the selfenergies of Mandelstam strings. $\mathcal{D}\nu_q(j_\mu^x)$ is the measure with the desired properties. This measure is supported on currents j_μ^x associated with q paths in a 3-plane at a fixed time starting at the site x and reaching infinity ("∞"). Comparing (11) and (10) we see that the measure $\mathcal{D}\nu_q(j_\mu^x)$ is peaked at E_μ^x at large scales. The 2-point correlation function for the gauge-invariant charged field in the compact abelian Higgs (c.H.) model is then given by

$$\int \mathcal{D}\nu_q(j_\mu^x) \int \mathcal{D}\nu_q(j_\mu^{x'}) \langle e^{i(\theta(x)-\theta(x'))} e^{ie\sum_{y,\mu}(j_\mu^x - j_\mu^{x'})(y) A_\mu(y)}\rangle_{c.H.} \tag{12}$$

replacing the r.h.s. of (5). This definition respects Dirac's quantization condition for fluxes and, as a consequence, it is independent of the Dirac strings of the magnetic monopoles of the compact Higgs model. [See [19] for preliminary numerical evidence for the validity of an order parameter for the Coulomb-Higgs transition in this model, based on the above correlation function.]

The 2-point monopole correlation function obtained by duality from (12) is given by

$$\int \mathcal{D}\nu_q(j_\mu^x) \int \mathcal{D}\nu_q(j_\mu^{x'}) \langle D(\Sigma(j^x - j^{x'} + j^{xx'}))\rangle \tag{13}$$

and plays the role of the l.h.s. of (5). Here $D(\Sigma)$ is the 't Hooft loop in the dual of the compact Higgs model corresponding to a surface Σ whose boundary is given by the support of $j^x - j^{x'} + j^{xx'}$, with b.c. turning it into a closed curve. $D(\Sigma)$ is obtained by shifting the field strength of A_μ by $2\pi q * \Sigma_{\mu\nu}$ in the action, where $q\Sigma_{\mu\nu}$ is the $\mathbf{Z}$-valued surface current supported on Σ. Since j_μ^x is supported on q paths, Σ is a q-sheet surface with the q sheets having a common boundary given by the single line support of $j_\mu^{xx'}$.

4. A new order parameter

To export these ideas to $SU(2)$ Yang-Mills theory, one first remarks that, in an abelian projection gauge, there appear a charged field, c, of electric charge 1 and monopoles of two species: i) $\mathbf{Z}_2$-singular monopoles with magnetic charge [20] $g = 2\pi$, whose worldlines are defined independently of the abelian projection. However they are screened [21] and dilute close to the continuum at $T = 0$ [3] and thus cannot induce confinement; ii) regular monopoles with magnetic charge $g = 4\pi$, whose worldlines are only defined within the abelian projection gauge. It is the condensation of these monopoles that should be responsible for confinement, and for them Dirac's quantization condition for charges is satisfied with $q = 2$.

Therefore, we propose [22] to construct the 2-point function for such regular monopoles, $G^{YM}(x, x')$, as in equation (13) for $q = 2$, with the following reinterpretation of notations: $\langle\cdot\rangle$ denotes the expectation value in $SU(2)$ Yang-Mills theory and $D(\Sigma)$ is the $SU(2)$-'t Hooft loop which is defined by replacing the plaquette term in (2) by

$$Tr\left(U_{\mu\nu}(y) e^{i\sigma_3 2\pi * \Sigma_{\mu\nu}(y)}\right). \tag{14}$$

$G^{YM}(x, x')$ is thus defined as a sum of 't Hooft loops, and the surfaces Σ involved have 2 connected boundaries, each at constant time, with fixed points the location of creation and annihilation, x and x', of the monopole. The definition of $G^{YM}(x, x')$ is clearly intrinsic to $SU(2)$ Yang-Mills theory, independent of the choice of an abelian projection. In an abelian projection

gauge, however, the surfaces Σ are viewed as 2-sheet surfaces of center vortices, with the two sheets joining along the support of $j^{xx'}$, which becomes the worldline of a regular monopole. Hence, whereas the definition of the worldline of a regular monopole necessitates the introduction of an abelian projection, the positions of creation and annihilation of the monopole are independent of it. There is no semiclassical analogue of such monopoles in the $SU(2)$ theory without abelian gauge fixing.

From correlation functions of regular monopole fields obtained generalizing in obvious way the above definition, one can reconstruct a monopole field operator $\hat{M}$. We claim that its v.e.v. is a good order parameter for the confinement-deconfinement transition. An argument supporting this conjecture goes as follows: Since $2\Sigma_{\mu\nu}$ is integer-valued, one can substitute σ_3 in (14) with any $su(2)$-valued field X of unit norm, selecting an abelian projection. Since the measure $\mathcal{D}\nu_2(j^x_\mu)$ is peaked near E^x, at large scales, one may argue (using that $B^x = *E^x$) that, in the scaling limit, $G^{YM}(x,x')$ behaves like the v.e.v. of the disorder operator (4) of [9] , which, numerically, is a good order parameter. By respecting the Dirac quantization condition for fluxes our construction of $G^{YM}(x,x')$ avoids the inconsistency in the treatment of small scales of previous monopole correlators and, although we expect that this inconsistency is irrelevant for the large distance behaviour controlling the phase transition, the independence of X of $G^{YM}(x,x')$ could explain why, numerically, the critical exponents of the transition have been found to be independent of the choice of abelian projection [16].

Finally, our construction, being based on center vortex sheets, points to a natural connection with the scenario of center dominance. To make this more concrete, we replace the $SU(2)$-field U_μ by a coset field $\bar{U}_\mu$, $SU(2)/\mathbf{Z}_2 \simeq SO(3)$-valued, and a $\mathbf{Z}_2 \simeq \{0,1\}$-valued 2-form $\sigma_{\mu\nu}$, obeying a constraint [3] which admits a gauge-dependent solution:

$$e^{i\pi\sigma_{\mu\nu}(y)} = sign[TrU_{\mu\nu}(y)]Z_{\mu\nu}(y) \qquad (15)$$

where $Z_{\mu\nu}(y)$ is the Wilson plaquette of $Z_\mu(y) = sign[TrU_\mu(y)]$. The 't Hooft disorder field $D(\Sigma)$ is obtained in terms of $\bar{U}_\mu$ and $\sigma_{\mu\nu}$ by shifting $\sigma_{\mu\nu}$ by $*2\Sigma_{\mu\nu}$ in the action (in the notation of (14)). Plaquettes with a value -1 for the first, gauge-invariant, term on the r.h.s. of (15) identify the support of thin vortices; a value -1 for the second term in the center projection gauge identifies the plaquettes in the support of P-vortices. There is numerical evidence [23] that P-vortex sheets are percolating in the space directions in the confinement phase, and this suggests that, in the center projection gauge, the introduction of the vortex sheets Σ, infinite in space directions, involved in the construction of monopole correlation functions should be a small perurbation, and the $\mathcal{D}\nu_2$ average of $\langle D(\Sigma)\rangle$ shoud not vanish, whence $\langle\hat{M}\rangle \neq 0$. In the deconfinement phase at positive temperature, however, P-vortex sheets appear, numerically, to be non-percolating in space directions [23], and one expects that the introduction of Σ then leads to clustering, implying $\langle\hat{M}\rangle$=0. Condensation of regular monopoles in the center projection gauge could then be interpreted as due to percolation in space directions of P-vortex sheets. The relation between worldlines of regular monopoles and vortex sheets, in our construction, is a natural extension to open worldlines, with boundaries corresponding to creation and annihilation of monopoles, of that appearing in [24] for closed monopole worldlines.

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