

# STRING THEORY, COSMOLOGY AND VARYING CONSTANTS

Thibault DAMOUR



Institut des Hautes Études Scientifiques  
35, route de Châtres  
91440 – Bures-sur-Yvette (France)

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# String theory, cosmology and varying constants \*

Thibault Damour

*Institut des Hautes Etudes Scientifiques, 35, route de Chartres, 91440  
Bures-sur-Yvette, France*

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**Abstract.** In string theory the coupling “constants” appearing in the low-energy effective Lagrangian are determined by the vacuum expectation values of some (a priori) massless scalar fields (dilaton, moduli). This naturally leads one to expect a correlated variation of all the coupling constants, and an associated violation of the equivalence principle. We review some string-inspired theoretical models which incorporate such a spacetime variation of coupling constants while remaining naturally compatible both with phenomenological constraints coming from geochemical data (Oklo; Rhenium decay) and with present equivalence principle tests. Barring a very unnatural fine-tuning of parameters, a variation of the fine-structure constant as large as that recently “observed” by Webb et al. in quasar absorption spectra appears to be incompatible with these phenomenological constraints. Independently of any model, it is emphasized that the best experimental probe of varying constants are high-precision tests of the universality of free fall, such as MICROSCOPE and STEP.

## 1. Introduction

String theory is the only known framework which reconciles perturbative gravity with quantum mechanics. Though string theory is not fully defined (beyond the perturbative level), and though there is no clear understanding of the way the Standard Model of particle physics eventually fits within this theory, it seems interesting to take seriously some of the hints suggested by the presently understood parts of string theory. One of these hints is that the coupling “constants” appearing in the low-energy Lagrangian are determined by the vacuum expectation values (VEV) of some a priori massless scalar fields: dilaton and moduli. The VEV of the dilaton  $\phi$  determines the basic string coupling constant  $g_s = e^{\phi/2}$  [1]. One can view this “softening” of all coupling constants as a generalization of Einstein’s theory of gravity. Before Einstein (1915), both the structure of spacetime and the laws of local matter interactions were supposed to be “rigid”, i.e. given once for all, as absolute structures, independently of the material content of the world. Einstein’s theory of general relativity introduced the idea

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that the structure of spacetime might be “soft”, i.e. influenced by its material content. However, he postulated (“principle of equivalence”) that the laws of local physics, and notably the values of all the (dimensionless) coupling constants ( $e^2/\hbar c$ ,  $m_e/m_p$ , ...), must be kept “rigidly fixed”. String theory treats symmetrically spacetime and matter interactions and suggests that both of them are “soft”. It is amusing to note that this generalized correlated “softening” of structures (which were traditionally considered as independent and rigid) serendipitously shows up even in the mathematical notation used to represent them, through a multiple use of the letter “ $g$ ”: at the tree-level of string theory there is a link not only between geometry and gravitation (through the unified geometrical field  $\mathbf{g}_{\mu\nu}(x)$ ), but also between gravitation ( $\mathbf{g}_{\mu\nu}(x)$ ), string coupling ( $\mathbf{g}_s(x)$ ), gauge couplings ( $\mathbf{g}(x)$ ) and gravitational coupling ( $\mathbf{G}(x)$ ). We refer here to a low-energy Lagrangian density of the form ( $\alpha' = 1$ )

$$\mathcal{L} = \sqrt{\tilde{g}} e^{-\phi} \left[ \tilde{R} + 2 \tilde{\square} \phi - (\tilde{\nabla} \phi)^2 - \frac{1}{4} \tilde{F}^2 + \dots \right]. \quad (1)$$

Actually such a tree-level Lagrangian (with a massless dilaton  $\phi$ ) is in conflict with experimental tests of the equivalence principle. Indeed, the dilaton has gravitational-strength couplings to matter which violate the equivalence principle [2, 3]. For instance, using the results of Ref. [3], one derives that the Lagrangian (1) predicts a violation of the universality of free fall at the level  $\Delta a/a \sim 10^{-5}$  (to be compared with the present limits  $\sim 10^{-12}$  [4]), and a time variation of the fine-structure constant  $e^2$  on cosmological scales:  $d \ln e^2 / dt \sim 10^{-10} \text{ yr}^{-1}$  (to be compared with the Oklo limit  $\sim 5 \times 10^{-17} \text{ yr}^{-1}$  [see below], or with laboratory limits  $\sim 10^{-14} \text{ yr}^{-1}$  [5, 6]).

It is generally assumed that this violent conflict with experimental tests of the equivalence principle is avoided because, after supersymmetry breaking, the dilaton<sup>1</sup> might acquire a (large enough) mass: say  $m_\phi \gtrsim 10^{-3} \text{ eV}$  so that observational deviations from Einstein’s gravity are quenched on distances larger than a fraction of a millimeter. If that were the case, there would also be no possibility to predict any time variation of the coupling constants on cosmological scales. There exists, however, a mechanism which can naturally reconcile a *massless* dilaton with existing experimental data: this is the *decoupling mechanism* of Ref. [3] (see also [7]). In the following, we shall review a recent work [8, 9] which has extended this mechanism in a manner which comes close to reconciling present experimental tests of the equivalence principle

<sup>1</sup> In the following, we use the word “dilaton” to denote the combination of the ten-dimensional dilaton and of various moduli which determines the values of the four-dimensional coupling constants.



with the recent results of Webb et al. [10] suggesting that the fine-structure constant  $e^2$  has varied by  $\sim 10^{-5}$  between redshifts of order 1 and now. For recent reviews of the flurry of works concerning the observational and theoretical aspects of the “variation of constants” see [11], and the other contributions to this workshop, notably [12].

## 2. Decoupling mechanism and dilaton runaway

The basic idea of Ref. [3] was to exploit the string-loop modifications of the (four dimensional) effective low-energy action (we use the signature  $-+++$ )

$$S = \int d^4x \sqrt{\tilde{g}} \left( \frac{B_g(\phi)}{\alpha'} \tilde{R} + \frac{B_\phi(\phi)}{\alpha'} [2 \tilde{\square} \phi - (\tilde{\nabla} \phi)^2] - \frac{1}{4} B_F(\phi) \tilde{F}^2 - \dots \right), \quad (2)$$

i.e. the  $\phi$ -dependence of the various coefficients  $B_i(\phi)$ ,  $i = g, \phi, F, \dots$ , given in the weak-coupling region ( $e^\phi \rightarrow 0$ ) by series of the form  $B_i(\phi) = e^{-\phi} + c_0^{(i)} + c_1^{(i)} e^\phi + c_2^{(i)} e^{2\phi} + \dots$ , coming from the genus expansion of string theory. It was shown in [3] that, if there exists a special value  $\phi_m$  of  $\phi$  which extremizes all the (relevant) coupling functions  $B_i^{-1}(\phi)$ , the cosmological evolution of the graviton-dilaton-matter system naturally drives  $\phi$  towards  $\phi_m$  (which is a fixed point of the Einstein-dilaton-matter system). This provides a mechanism for fixing a massless dilaton at a value where it *decouples* from matter (“Least Coupling Principle”). Refs. [8, 9] considered the case (recently suggested in [13]) where the coupling functions, at least in the visible sector, have a smooth *finite* limit for infinite bare string coupling  $g_s \rightarrow \infty$ . In this case, quite generically, we expect

$$B_i(\phi) = C_i + \mathcal{O}(e^{-\phi}). \quad (3)$$

Under this assumption, the coupling functions are all extremized at infinity, i.e. a fixed point of the cosmological evolution is  $\phi_m = +\infty$ . [See [14] for an exploration of the late-time cosmology of models satisfying (3).] It was found that the “decoupling” of such a “run-away” dilaton has remarkable features: (i) the residual dilaton couplings at the present epoch can be related to the amplitude of density fluctuations generated during inflation, and (ii) these residual couplings, while being naturally compatible with present experimental data, are predicted to be large enough to be detectable by a modest improvement in the precision of equivalence principle tests (non universality of the free fall, and,

possibly, variation of “constants”). This result contrasts with the case of attraction towards a finite value  $\phi_m$  which leads to extremely small residual couplings [15].

One assumes some primordial inflationary stage driven by the potential energy of an inflaton field  $\chi$ . Working with the Einstein frame metric  $g_{\mu\nu} = C_g^{-1} B_g(\phi) \tilde{g}_{\mu\nu}$ , and with the modified dilaton field  $\varphi = \int d\phi[(3/4)(B'_g/B_g)^2 + B'_\phi/B_g + (1/2) B_\phi/B_g]^{1/2}$ , one considers an effective action of the form

$$S = \int d^4x \sqrt{g} \left[ \frac{\tilde{m}_P^2}{4} R - \frac{\tilde{m}_P^2}{2} (\nabla\varphi)^2 - \frac{\tilde{m}_P^2}{2} F(\varphi) (\nabla\chi)^2 - \tilde{m}_P^4 V(\chi, \varphi) \right], \quad (4)$$

where  $\tilde{m}_P^2 = 1/(4\pi G) = 4C_g/\alpha'$ , and where the dilaton dependence of the Einstein-frame action is related to its (generic) string-frame dependence (2) by  $F(\varphi) = B_\chi(\phi)/B_g(\phi)$ ,  $V(\chi, \varphi) = C_g^2 \tilde{m}_P^{-4} B_g^{-2}(\phi) \tilde{V}(\tilde{\chi}, \phi)$ .

Under the basic assumption (3),  $d\varphi/d\phi$  tends, in the strong-coupling limit  $\phi \rightarrow +\infty$ , to the constant  $1/c$ , with  $c \equiv (2C_g/C_\phi)^{1/2}$ , so that the asymptotic behaviour of the bare string coupling is

$$g_s^2 = e^\phi \simeq e^{c\varphi}. \quad (5)$$

Let us consider for simplicity the case where  $F(\varphi) = 1$  and  $V(\chi, \varphi) = \lambda(\varphi) \chi^n/n$  with a dilaton-dependent inflaton coupling constant  $\lambda(\varphi)$  of the form

$$\lambda(\varphi) = \lambda_\infty (1 + b_\lambda e^{-c\varphi}), \quad (6)$$

where we assume that  $b_\lambda > 0$ , i.e that  $\lambda(\varphi)$  reaches a *minimum* at strong-coupling,  $\varphi \rightarrow +\infty$ . It is shown in [9] that this simple case is representative of rather general cases of  $\varphi$ -dependent inflationary potentials  $V(\chi, \varphi)$ .

During inflation ( $ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j$ ), it is easily seen that, while  $\chi$  slowly rolls down towards  $\chi \sim 1$ , the dilaton  $\varphi$  is monotonically driven towards large values. The solution of the (classical) slow-roll evolution equations leads to

$$e^{c\varphi} + \frac{b_\lambda c^2}{2n} \chi^2 = \text{const.} = e^{c\varphi_{\text{in}}} + \frac{b_\lambda c^2}{2n} \chi_{\text{in}}^2. \quad (7)$$

Using the result (7), one can estimate the value  $\varphi_{\text{end}}$  of  $\varphi$  at the end of inflation by inserting for the initial value  $\chi_{\text{in}}$  of the inflaton the value corresponding to the end of self-regenerating inflation [16]. One remarks that the latter value can be related to the amplitude  $\delta_H \sim 5 \times 10^{-5}$

of density fluctuations, on the scale corresponding to our present horizon, generated by inflation, through  $\chi_{\text{in}} \simeq 5\sqrt{n} (\delta_H)^{-2/(n+2)}$ . Finally, assuming  $e^{c\varphi_{\text{in}}} \ll e^{c\varphi_{\text{end}}}$ , one gets the estimate:

$$e^{c\varphi_{\text{end}}} \sim 12.5c^2 b_\lambda (\delta_H)^{-\frac{4}{n+2}}. \quad (8)$$

A more general study [9] of the run-away of the dilaton during inflation (including an estimate of the effect of quantum fluctuations) only modifies this result by a factor  $\mathcal{O}(1)$ . It is also found that the present value of the dilaton is well approximated by  $\varphi_{\text{end}}$ .

### 3. Deviations from general relativity induced by a runaway dilaton

Eq. (8) tells us that, within this scenario, the smallness of the present matter couplings of the dilaton is quantitatively linked to the smallness of the (horizon-scale) cosmological density fluctuations. To be more precise, and to study the compatibility with present experimental data, one needs to estimate the crucial dimensionless quantity

$$\alpha_A(\varphi) \equiv \partial \ln m_A(\varphi) / \partial \varphi, \quad (9)$$

which measures the coupling of  $\varphi$  to a massive particle of type  $A$ . The definition of  $\alpha_A$  is such that, at the Newtonian approximation, the interaction potential between particle  $A$  and particle  $B$  is  $-G_{AB} m_A m_B / r_{AB}$  where [7, 3]  $G_{AB} = G(1 + \alpha_A \alpha_B)$ . Here,  $G$  is the bare gravitational coupling constant entering the Einstein-frame action (4), and the term  $\alpha_A \alpha_B$  comes from the additional attractive effect of dilaton exchange.

Let us first consider the (approximately) *composition-independent* deviations from general relativity, i.e. those that do not essentially depend on violations of the equivalence principle. Most composition-independent gravitational experiments (in the solar system or in binary pulsars) consider the long-range interaction between objects whose masses are essentially baryonic (the Sun, planets, neutron stars). As argued in [2, 3] the relevant coupling coefficient  $\alpha_A$  is then approximately universal and given by the logarithmic derivative of the QCD confinement scale  $\Lambda_{\text{QCD}}(\varphi)$ , because the mass of hadrons is essentially given by a pure number times  $\Lambda_{\text{QCD}}(\varphi)$ . [We shall consider below the small, non-universal, corrections to  $m_A(\varphi)$  and  $\alpha_A(\varphi)$  linked to QED effects and quark masses.] Remembering from Eq. (2) the fact that, in the string frame (where there is a fixed cut-off linked to the string mass  $\widetilde{M}_s \sim (\alpha')^{-1/2}$ ) the gauge coupling is dilaton-dependent ( $g_F^{-2} =$



$B_F(\varphi)$ ), we see that (after conformal transformation) the Einstein-frame confinement scale has a dilaton-dependence of the form

$$\Lambda_{\text{QCD}}(\varphi) \sim C_g^{1/2} B_g^{-1/2}(\varphi) \exp[-8\pi^2 b_3^{-1} B_F(\varphi)] \widetilde{M}_s, \quad (10)$$

where  $b_3$  denotes the one-loop (rational) coefficient entering the Renormalization Group running of  $g_F$ . Here  $B_F(\varphi)$  denotes the coupling to the SU(3) gauge fields. For simplicity, we shall assume that (modulo rational coefficients) all gauge fields couple (near the string cut off) to the same  $B_F(\varphi)$ . Such an assumption is natural in a stringy framework. Note that we differ here from the line of work of Bekenstein [17], recently extended in [18, 19], which assumes that  $\varphi$  couples *only* to the electromagnetic gauge field. The string-inspired assumption of coupling to all gauge fields yields the following approximately universal dilaton coupling to hadronic matter

$$\alpha_{\text{had}}(\varphi) \simeq \left( \ln \left( \frac{\widetilde{M}_s}{\Lambda_{\text{QCD}}} \right) + \frac{1}{2} \right) \frac{\partial \ln B_F^{-1}(\varphi)}{\partial \varphi}. \quad (11)$$

Numerically, the coefficient in front of the R.H.S. of (11) is of order 40. Consistently with the basic assumption (3), one parametrizes the  $\varphi$  dependence of the gauge coupling  $g_F^2 = B_F^{-1}$  as

$$B_F^{-1}(\varphi) = B_F^{-1}(+\infty) [1 - b_F e^{-c\varphi}]. \quad (12)$$

We finally obtain

$$\alpha_{\text{had}}(\varphi) \simeq 40 b_F c e^{-c\varphi}. \quad (13)$$

Inserting the estimate (8) of the value of  $\varphi$  reached because of the cosmological evolution, one gets the estimate

$$\alpha_{\text{had}}(\varphi_{\text{end}}) \simeq 3.2 \frac{b_F}{b_\lambda c} \delta_H^{\frac{4}{n+2}}. \quad (14)$$

It is plausible to expect that the quantity  $c$  (which is a ratio) and the ratio  $b_F/b_\lambda$  are both of order unity. This then leads to the numerical estimate  $\alpha_{\text{had}}^2 \sim 10 \delta_H^{\frac{8}{n+2}}$ , with  $\delta_H \simeq 5 \times 10^{-5}$ . An interesting aspect of this result is that the expected present value of  $\alpha_{\text{had}}^2$  depends rather strongly on the value of the exponent  $n$  (which entered the inflaton potential  $V(\chi) \propto \chi^n$ ). In the case  $n = 2$  (i.e.  $V(\chi) = \frac{1}{2} m_\chi^2 \chi^2$ ) we have  $\alpha_{\text{had}}^2 \sim 2.5 \times 10^{-8}$ , while if  $n = 4$  ( $V(\chi) = \frac{1}{4} \lambda \chi^4$ ) we have  $\alpha_{\text{had}}^2 \sim 1.8 \times 10^{-5}$ . Both estimates are compatible with present (composition-independent) experimental limits on deviations from Einstein's theory (in the solar system, and in binary pulsars). For instance, the ‘‘Eddington’’ parameter  $\gamma - 1 \simeq -2 \alpha_{\text{had}}^2$  is compatible with the

present best limits  $|\gamma - 1| \lesssim 2 \times 10^{-4}$  coming from Very Long Baseline Interferometry measurements of the deflection of radio waves by the Sun [20].

Let us consider situations where the non-universal couplings of the dilaton induce (apparent) *violations of the equivalence principle*. This means considering the composition-dependence of the dilaton coupling  $\alpha_A$ , Eq. (9), i.e. the dependence of  $\alpha_A$  on the type of matter we consider. Two test masses, made respectively of  $A$ - and  $B$ -type particles will fall in the gravitational field generated by an external mass  $m_E$  with accelerations differing by

$$\left(\frac{\Delta a}{a}\right)_{AB} \equiv 2 \frac{a_A - a_B}{a_A + a_B} \simeq (\alpha_A - \alpha_B) \alpha_E. \quad (15)$$

We have seen above that in lowest approximation  $\alpha_A \simeq \alpha_{\text{had}}$  does not depend on the composition of  $A$ . We need, however, now to retain the small composition-dependent effects to  $\alpha_A$  linked to the  $\varphi$ -dependence of QED and quark contributions to  $m_A$ . This has been investigated in [3] with the result that  $\alpha_A - \alpha_{\text{had}}$  depends linearly on the baryon number  $B \equiv N + Z$ , the neutron excess  $D \equiv N - Z$ , and the quantity  $E \equiv Z(Z - 1)/(N + Z)^{1/3}$  linked to nuclear Coulomb effects. Under the assumption that the latter dependence is dominant, and using the average estimate  $\Delta(E/M) \simeq 2.6$  (applicable to mass pairs such as (Beryllium, Copper) or (Platinum, Titanium)), one finds that the violation of the universality of free fall is approximately given by

$$\left(\frac{\Delta a}{a}\right) \simeq 5.2 \times 10^{-5} \alpha_{\text{had}}^2 \simeq 5.2 \times 10^{-4} \left(\frac{b_F}{b_\lambda c}\right)^2 \delta_H^{\frac{8}{n+2}}. \quad (16)$$

This result is one of the main predictions of the present model. If one inserts the observed density fluctuation  $\delta_H \sim 5 \times 10^{-5}$ , one obtains a level of violation of the universality of free fall (UFF) due to a runaway dilaton which is  $\Delta a/a \simeq 1.3(b_F/(b_\lambda c))^2 \times 10^{-12}$  for  $n = 2$  (i.e. for the simplest chaotic inflationary potential  $V(\chi) = \frac{1}{2} m_\chi^2(\phi) \chi^2$ ), and  $\Delta a/a \simeq 0.98(b_F/(b_\lambda c))^2 \times 10^{-9}$  for  $n = 4$  (i.e. for  $V(\chi) = \frac{1}{4} \lambda(\phi) \chi^4$ ). The former case is naturally compatible with current tests (at the  $\sim 10^{-12}$  level [4]) of the UFF. Values  $n \geq 4$  of the exponent require (within this scenario) that the (unknown) dimensionless combination of parameters  $(b_F/(b_\lambda c))^2$  be significantly smaller than one.



#### 4. Cosmological variation of “constants”

Let us also consider another possible deviation from general relativity and the standard model: a possible variation of the coupling constants, most notably of the fine structure constant  $e^2/\hbar c$  on which the strongest limits are available. Consistently with our previous assumptions we expect  $e^2 \propto B_F^{-1}(\varphi)$  so that, from (12),  $e^2(\varphi) = e^2(+\infty) [1 - b_F e^{-c\varphi}]$ . The logarithmic variation of  $e^2$  (introducing the derivative  $\varphi' = d\varphi/dp$  with respect to the “e-fold” parameter  $dp = H dt = da/a$ ) is thus given by

$$\frac{d \ln e^2}{H dt} \simeq b_F c e^{-c\varphi} \varphi' \simeq \frac{1}{40} \alpha_{\text{had}} \varphi'. \quad (17)$$

The value of  $\varphi'$  depends on the coupling of the dilaton to the two currently dominating energy forms in the universe: dark matter (coupling  $\alpha_m(\varphi)$ ), and vacuum energy (coupling  $\alpha_V = \frac{1}{4} \partial \ln V(\varphi) / \partial \varphi$ ). In the slow-roll approximation, the cosmological evolution of  $\varphi$  is given by

$$(\Omega_m + 2\Omega_V) \varphi' = -\Omega_m \alpha_m - 4\Omega_V \alpha_V, \quad (18)$$

where  $\Omega_m$  and  $\Omega_V$  are, respectively, the dark-matter- and the vacuum-fraction of critical energy density ( $\rho_c \equiv (3/2) \tilde{m}_P^2 H^2$ ). The precise value of  $\varphi'$  is model-dependent and can vary (depending under the assumptions one makes) from an exponentially small value ( $\varphi' \sim e^{-c\varphi}$ ) to a value of order unity. In models where either the dilaton is more strongly coupled to dark matter than to ordinary matter [21], or/and plays the role of quintessence (as suggested in [14]),  $\varphi'$  can be of order unity. Assuming just spatial flatness and saturation of the “energy budget” by non-relativistic matter and dilatonic quintessence, one can relate the value of  $\varphi' = d\varphi/(H dt)$  to  $\Omega_m$  and to the deceleration parameter  $q \equiv -\ddot{a}a/\dot{a}^2$ :

$$\varphi'^2 = 1 + q - \frac{3}{2} \Omega_m. \quad (19)$$

The supernovae Ia data [22] give a strict upper bound on the present value  $q_0$ :  $q_0 < 0$ . A generous lower bound on the present value of  $\Omega_m$  is  $\Omega_{m0} > 0.2$  [23]. Inserting these two constraints in Eq.(19) finally yields the safe upper bound on the current value of  $\varphi'$

$$\varphi_0'^2 < 0.7, \text{ i.e. } |\varphi_0'| < 0.84. \quad (20)$$

On the other hand, Eq. (16) yields the link

$$\alpha_{\text{had}} \simeq \pm 1.4 \times 10^{-4} \sqrt{10^{12} \frac{\Delta a}{a}}. \quad (21)$$

Inserting this result in Eq. (17) yields

$$\frac{d \ln e^2}{H dt} \simeq \pm 3.5 \times 10^{-6} \sqrt{10^{12} \frac{\Delta a}{a}} \varphi', \quad (22)$$

which yields, upon integration over  $p = \ln a + \text{cst} = -\ln(1+z) + \text{cst}$ ,

$$\frac{\Delta e^2}{e^2} \equiv \frac{e^2(z) - e^2(0)}{e^2(0)} \simeq \mp 3.5 \times 10^{-6} \sqrt{10^{12} \frac{\Delta a}{a}} \langle \varphi' \rangle_z \ln(1+z), \quad (23)$$

where  $\langle \varphi' \rangle_z \equiv (\varphi(p) - \varphi(p_0))/(p - p_0)$  denotes the average value of  $\varphi'$  between now and redshift  $z$ . If we insert in Eq. (22) the limit  $\Delta a/a \lesssim 10^{-12}$ , coming from present experimental tests of the universality of free fall (UFF) [4], as well as the cosmological constraint (20) on the present value of  $\varphi'$ , we find that the present variation of the fine-structure constant is constrained to be  $|d \ln e^2 / H dt| \lesssim 3 \times 10^{-6}$ , i.e.  $|d \ln e^2 / dt| \lesssim 2 \times 10^{-16} \text{ yr}^{-1}$ . Such a level of variation is comparable to the planned sensitivity of currently developed cold-atom clocks [6].

However, there are stronger constraints coming from geochemical data. Let us first recall that a *secure* limit on the time variation of  $e^2$  coming from the Oklo phenomenon is  $|\Delta e^2 / e^2| \lesssim 10^{-7}$  between now and  $\sim 2 \text{ Gyr}$  ago, i.e.  $|d \ln e^2 / dt| \lesssim 5 \times 10^{-17} \text{ yr}^{-1}$ . This limit was obtained in [24] under two very conservative assumptions: 1. A conservative interpretation of Oklo data making minimal assumptions about the temperature of the reaction zone, and about the possible amplitude of variation of the resonance energy  $E_r$  of the relevant excited state of Samarium 150, and 2. An analysis of the  $e^2$ -variation of the latter resonance energy  $E_r$  taking into account only the (rather well-known) Coulomb effects. We note that Ref. [25] derived a stronger limit on the variation of  $e^2$  by replacing assumption 1. above by the non-conservative assumption that  $E_r$  has stayed close to its present value. We note also that Ref. [26] derived a stronger limit on the variation of  $e^2$  by relaxing assumption 2. and by trying to estimate the indirect  $e^2$ -dependence of  $E_r$  coming from light quark contributions to the nuclear binding energy.

It is important to note that Ref. [26] also derived the strong limit  $|\Delta e^2 / e^2| \lesssim 3 \times 10^{-7}$  between now and 4.6 Gyr ago. This limit was derived from the Rhenium/Osmium ratio in 4.6 Gyr old iron-rich meteorites by making quite conservative assumptions. In particular, we note that, similarly to the Oklo assumption 2. above, this limit uses only the Coulomb effects in the  $\beta$ -decay rate of Rhenium 187. [Ref. [26] quotes also stronger limits obtained by trying to estimate the indirect  $e^2$ -dependence of the latter  $\beta$ -decay rate.]



Using Eq. (23), the conservative “Oklo” and “Rhenium” limits on the variation of  $e^2$ , corresponding to redshifts  $z_{\text{Oklo}} \simeq 0.15$  and  $z_{\text{Re}} \simeq 0.45$ , can be converted into constraints on the product  $r \langle \varphi' \rangle_z$  where  $r \equiv \sqrt{10^{12} \Delta a / a}$ . In round numbers, one finds that these two constraints read

$$\sqrt{10^{12} \frac{\Delta a}{a}} |\langle \varphi' \rangle_{z \simeq 0.15}| \lesssim 0.2; \quad \sqrt{10^{12} \frac{\Delta a}{a}} |\langle \varphi' \rangle_{z \simeq 0.45}| \lesssim 0.2. \quad (24)$$

As seen on Eq. (18), the cosmological evolution of  $\varphi$  is driven by two quantities: the coupling  $\alpha_m$  of  $\varphi$  to dark matter, and its coupling  $\alpha_V$  to vacuum energy. If one had only the Oklo constraint to cope with, one could fine-tune the *ratio*  $\alpha_m / \alpha_V$  to satisfy the first constraint (24) without constraining the overall magnitudes of  $\alpha_m$  and  $\alpha_V$ . This is what was done in Ref. [19] (within the different context of Bekenstein-like models) to exhibit models satisfying the Oklo and UFF constraints and allowing for a variation of  $e^2$  around  $z \sim 1$  driven by a large enough  $\alpha_m$  (à la [27]) to explain the observational results of Webb et al. [10]. [Note that the implementation of the same idea within the context of dilaton-like models [8, 9] leads to a maximal possible variation of  $e^2$  which falls short, by a factor  $\sim 4$ , of the level needed to explain the results of [10].] However, the new point we wish to emphasize here is that the recently obtained “Rhenium constraint” [26], i.e. the second inequality (24), makes it impossible to concoct a fine-tuned ratio  $\alpha_m / \alpha_V$  so as to satisfy the two geochemical constraints (24), which correspond to quite different redshifts and therefore to significantly different relative weights  $\Omega_m \propto (1+z)^3$  and  $\Omega_V \propto (1+z)^0$ . If we were to consider more complicated models, in particular models where the “kinetic term”  $\propto \varphi''$  must be included in Eq. (18), and allows<sup>†</sup> for an oscillatory behaviour (as in local-attractor models [3]), it might be possible to fine-tune more parameters (like the phase of oscillation of  $\varphi$ ) so as to satisfy the two constraints (24). However, such a heavy fine-tuning becomes very unnatural. The most natural conclusion is that both  $\alpha_m$  and  $\alpha_V$  must be small enough, so that the quantity  $\sqrt{10^{12} \Delta a / a} |\langle \varphi' \rangle_z|$  is smaller than 0.2 for all redshifts where the cosmologically dominant energy forms are dark matter and/or dark energy. Eq. (23) then leads to the constraint

$$\frac{|e^2(z) - e^2(0)|}{e^2(0)} \lesssim 0.7 \times 10^{-6} \ln(1+z). \quad (25)$$

Note that this constraint is about ten times smaller than the claim of [10].



## 5. Conclusions

A first conclusion is that the results of Webb et al. [10] cannot be naturally explained within any model where the time variation of the fine-structure constant  $e^2$  is driven by the spacetime variation of a very light scalar field  $\varphi$ . On the other hand, the recently explored [8, 9] class of dilaton-like models with an attractor “at infinity” in field space naturally predicts the existence of small, but not unmeasurably small, violations of the equivalence principle. In the case where the dilaton  $\varphi$  is more significantly coupled to dark matter and/or dark energy than to ordinary matter, dilaton-like models can lead to a cosmological variation of  $e^2$  as large as Eq. (25). [We recall that this upper bound takes into account the two stringent geochemical bounds on  $\Delta e^2/e^2$  coming from the conservative interpretations of Oklo data [24] and of Rhenium decay data [23].] A time variation of the order of the upper limit in Eq. (25) might be observable through the comparison of high-accuracy cold-atom clocks. It might also be observable in astronomical spectral data (if one understands how to explain and subtract the systematic effects leading to the current apparent variation of  $e^2$  at the level  $\Delta e^2/e^2 \simeq -0.7 \times 10^{-5}$  [10]).

Finally, an important conclusion of all theoretical models where the time variation of  $e^2$  is linked to the spacetime variation of a light scalar field  $\varphi$  (be it the dilaton of string theory [3, 8, 9] or a field constrained to couple only to electromagnetism [17, 18, 19]) is that a *necessary* condition for having a fractional variation of  $e^2$  larger than  $10^{-6}$  on cosmological time scales is to have a violation of the universality of free fall (UFF) larger than about  $10^{-13}$  (see Eq. (22) in which  $|\varphi'|$  is certainly constrained by cosmological data to be smaller than 1, as discussed in [9]). Note that a measurably large violation of the UFF is a necessary, but by no means sufficient, condition for having a measurably large cosmological variation of  $e^2$ . Indeed, in Eq. (22) or Eq. (23) the value of  $\varphi' \equiv d\varphi/(Hdt)$  depends on the strength of the coupling of  $\varphi$  to the dominant forms of energy in the universe:  $\alpha_m$  and  $\alpha_V$ . These quantities could well be comparable to the strength of the coupling of  $\varphi$  to hadronic matter,  $\alpha_{\text{had}}$ , which is constrained to be small by UFF experiments (see Eq. (21)). This shows that the best experimental probe of an eventual “variation of constants” is to probe their *spatial* variation through high-precision tests of the UFF, rather than their (cosmological) time variation (see also [28] for a detailed discussion of clock experiments). This gives additional motivation for improved tests of the UFF, such as the Centre National d’Etudes Spatiales (CNES) mission MICROSCOPE [29] (to fly in 2004; planned sensitivity:  $\Delta a/a \sim 10^{-15}$ ), and the National Aeronautics and

Space Agency (NASA) and European Space Agency (ESA) mission STEP (Satellite Test of the Equivalence Principle; planned sensitivity:  $\Delta a/a \sim 10^{-18}$ ) [30].

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