GENERAL RELATIVITY AS A COSMOLOGICAL ATTRACTOR OF TENSOR-SCALAR THEORIES

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Décembre 1992

IHES/P/92/94

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ABSTRACT

Tensor-scalar theories of gravity are shown to generically contain an attractor mechanism toward general relativity, with the redshift at the beginning of the matter-dominated era providing the measure for the present level of deviation from general relativity. Quantitative estimates for the post-Newtonian parameters $\gamma - 1$, $\beta - 1$ and $\dot{G}/G$ are given, which give greater significance to future improvements of solar-system gravitational tests.

PACS : 04.80.+z, 04.50.+h, 98.80.Cq, 98.80.Dr

Most attempts at unifying gravity with the other interactions (from the original Kaluza-Klein theory down to superstring theories) include a prediction of the existence of massless scalar fields coupled to matter with gravitational strength. An independent motivation for considering scalar partners to the usual tensor gravity of Einstein is furnished by inflationary models which find in the framework of tensor-scalar theories of gravity a technically natural way of exiting inflation [1–3]. In a fundamental tensor-scalar theory one expects the ratio α^2 between the couplings to matter of scalar and tensor fields to be of order unity. However, the present solar-system gravitational experiments set (at the 1σ confidence level) a tight upper bound on this ratio [4],

$$\alpha^2_{\text{solar-system}} < 10^{-3} \ , \tag{1}$$

which seems to argue against the existence of long-range scalars. Perhaps such a pessimistic interpretation of the limit (1) is premature.

By examining general tensor-scalar cosmological models, we find that they generically contain an attractor mechanism toward general relativity, i.e. α^2 tends toward zero during the matter-dominated era of cosmological evolution. The possibility of such a mechanism has been previously suggested [2, 3], but without giving any firm argument that general relativity is indeed a generic attractor of tensor-scalar theories, nor quantitative estimates of the efficiency of this attractor mechanism.

The most general action describing a metric tensor-scalar theory (with massless fields) is [5–7] (see Ref. [8] for the general tensor-multi-scalar case)

$$S = (16\pi G_*)^{-1} \int d^4x\, g_*^{1/2} [R_* - 2\, g_*^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi] + S_m[\psi_m,\, A^2(\varphi) g^*_{\mu\nu}] \; . \tag{2}$$

G_* denotes a bare gravitational coupling constant, and $R_* \equiv g_*^{\mu\nu} R^*_{\mu\nu}$ the curvature scalar of the "Einstein metric" $g^*_{\mu\nu}$. The last term in Eq. (2) denotes the action of the matter, which is a functional of some matter variables, collectively denoted by ψ_m, and of the ("Jordan-Fierz") metric

$$\tilde{g}_{\mu\nu} \equiv A^2(\varphi) g^*_{\mu\nu} \; .$$

The universal coupling of matter to $\tilde{g}_{\mu\nu}$ means that physical rods and clocks measure this metric. However, the field equations of the theory (and in particular the cosmological evolution equations) are better formulated in terms of the variables $(g^*_{\mu\nu}, \varphi)$ which describe the two types of dynamical degrees of freedom present in the theory (massless helicity–2 and massless helicity–0 excitations). The logarithm of the conformal factor relating $\tilde{g}_{\mu\nu}$ to $g^*_{\mu\nu}$,

$$a(\varphi) \equiv \ln A(\varphi) \; ,$$

and its gradient,

$$\alpha(\varphi) \equiv \partial a(\varphi) / \partial \varphi \; ,$$

are the two basic functions describing the coupling between the canonical scalar field and matter. [$A \equiv \exp a$ and α are linked to the traditional Jordan-Fierz-Brans-Dicke field Φ and its associated (field-dependent) parameter $\omega(\Phi)$ through: $G_* A^2 = \Phi^{-1}$, $\alpha^2 = (2\omega + 3)^{-1}$].

The field equations read

$$R^*_{\mu\nu} = 2\partial_\mu \varphi\, \partial_\nu \varphi + 8\pi G_* \left(T^*_{\mu\nu} - \frac{1}{2} T^* g^*_{\mu\nu} \right) \; , \tag{3a}$$

$$\Box_{g_*} \varphi = -4\pi G_* \alpha(\varphi) T_* \; , \tag{3b}$$

with $T_*^{\mu\nu} \equiv 2(g_*)^{-1/2} \delta S_m / \delta g^*_{\mu\nu}$ denoting the stress-energy tensor in the g^* units, and where all tensorial operations are performed by using this metric. The gradient $\alpha(\varphi)$ in Eq. (3b) plays the rôle of the basic coupling strength between the scalar field and matter. Its square α^2 appears in all quantities where a scalar interaction mediates between two bodies. Its present value is constrained by Eq. (1).

Homogeneous cosmological spacetimes can be represented both in the Einstein conformal frame,

$$ds_*^2 = -dt_*^2 + R_*^2(t_*)d\ell^2 \ ,$$

and in the Jordan-Fierz one,

$$d\tilde{s}^2 = -d\tilde{t}^2 + \widetilde{R}^2(\tilde{t})d\ell^2 \ ,$$

with

$$d\ell^2 = (1 - kr^2)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

denoting the metric of a 3–space of constant curvature $k = +1, 0$ or -1. The physical cosmic time $\tilde{t}$ and scale factor $\widetilde{R}$ are related to their Einstein-frame counterparts through

$$d\tilde{t} = A(\varphi(t_*))dt_* \ , \quad \widetilde{R}(\tilde{t}) = A(\varphi(t_*))R_*(t_*) \ ,$$

in which $\varphi(t_*)$ is the (spatially averaged) cosmological value of the scalar field. The field equations (3) give (with an overdot denoting d/dt_*, and $H_* \equiv \dot{R}_*/R_*$)

$$-3\ddot{R}_*/R_* = 4\pi G_*(\rho_* + 3p_*) + 2\dot{\varphi}^2 \ , \tag{4a}$$

$$3H_*^2 + 3k/R_*^2 = 8\pi G_*\rho_* + \dot{\varphi}^2 \ , \tag{4b}$$

$$\ddot{\varphi} + 3H_*\dot{\varphi} = -4\pi G_*(\rho_* - 3p_*)\alpha(\varphi) \ . \tag{4c}$$

The g^*–frame density and pressure ($T_*^{\mu\nu} = (\rho_* + p_*)\, u_*^\mu u_*^\nu + p_* g_*^{\mu\nu}$, with $g_{\mu\nu}^* u_*^\mu u_*^\nu = -1$) are related to their directly measurable counterparts by $\rho_* = A^4\tilde{\rho}$, $p_* = A^4\tilde{p}$.

We found that it was possible in Eqs. (4) to decouple the cosmological evolution of the scalar field by introducing as evolution parameter the "p time" (not to be confused with the pressures p_* or $\tilde{p}$):

$$p = \int H_* dt_* = \ln\, R_* + \text{ const.}$$

For simplicity we shall here restrict ourselves to spatially flat cosmologies, $k = 0$ [see Ref. [9] for the discussion of the general case]. Then Eqs. (4) yields the following simple decoupled equation for the p–evolution of φ (with a prime denoting d/dp)

$$\frac{2}{3 - \varphi'^2}\varphi'' + (1 - \lambda)\varphi' = -(1 - 3\lambda)\alpha(\varphi) \ , \tag{5}$$

with $\lambda \equiv p_*/\rho_* \equiv \tilde{p}/\tilde{\rho}$ being, under the usual approximations, a known numerical constant during each cosmological era: $\lambda = -1,\ 1/3,\ 0$ in the inflationary, radiative and matter-dominated eras, respectively.

The qualitative features of the dynamics described by Eq. (5) is readily seen by its mechanical analog: motion (in "p" time) along the φ-axis of a particle with velocity-dependent inertial mass $m(\varphi') = 2/(3 - \varphi'^2)$ experiencing a damping force $-(1 - \lambda)\varphi'$ and an external force deriving from the potential $+(1 - 3\lambda)a(\varphi)$. [The positivity of $\tilde{\rho}$ implies $\varphi'^2 < 3$; Eq. (5) represents a kind of "relativistic" dynamics]. During the radiation era ($\lambda = 1/3$) φ is decoupled from the external potential and any initial velocity φ' brought into this era is quickly (within a few units of p time) damped out, and φ comes to rest, $\varphi = \varphi_R =$ const.. The scalar field's evolution in the subsequent matter era ($\lambda = 0$) is then the motion of a particle starting at rest somewhere in the potential $a(\varphi) \equiv \ln A(\varphi)$ which responds to the gradient of the potential and is subject to simple damping. Therefore φ simply moves down any gradient of $a(\varphi)$ and tends to be captured (by damping) near a minimum of $a(\varphi)$ (where $\alpha(\varphi_{\rm min}) = \partial a/\partial\varphi_{\rm min} = 0$). This precisely describes an attractor mechanism toward general relativity ($\alpha = 0$), which can be expected to take place for generic classes of coupling functions $a(\varphi)$ and starting values φ_R of φ out of the radiation era. This naturally reconciles a fundamental tensor-scalar theory of gravity with the tight experimental upper bound (1). [Among the exceptions that do not belong to the class of GR-attracted theories, one can note the original Jordan-Fierz theory ($a(\varphi) = \alpha\varphi$; constant-slope potential), and the constant-G theory [10] ($a(\varphi) = \ln\cos\varphi$ in which φ tends to fall all the way down to $a = -\infty$, $|\alpha| = +\infty$; i.e. toward pure scalar gravity)].

An estimate of the efficiency of this attractor mechanism can be made by considering the simple model of a parabolic (attractive) potential, $a(\varphi) = \frac{1}{2}\kappa\varphi^2$. With sufficient curvature of the potential ($\kappa > 3/8$), φ undergoes damped oscillatory motion about the minimum of $a(\varphi)$,

$$\varphi(p) = \frac{\alpha_R}{\kappa}\left(1 - \frac{3}{8\kappa}\right)^{-1/2} e^{-\frac{3}{4}p} \sin(\omega p + \theta_R) \ , \tag{6}$$

with $\omega \equiv (3/4)(8\kappa/3 - 1)^{1/2}$, $\theta_R \equiv \tan^{-1}[(8\kappa/3 - 1)^{1/2}]$, and $\alpha_R = \kappa\varphi_R$ being the slope of the potential at the initial position of φ (i.e. at the end of the radiation era).

If inflation did not occur, it is natural to assume that the state of the tensor-scalar system coming out of some primordial Planck era was order-of-unity away from general relativity; then $\alpha_R =$ const. ~ 1 during the subsequent radiation era. On the other hand, if inflation did occur, it has been found [11, 12] that the slope α_R upon entering the radiation era must be greater than some lower-bound $\alpha_{\rm min}$ (estimated to be at least 0.16 [11]) in order that inflation terminate adequately. If this happens in a non fine-tuned way [13], we expect to have again $\alpha_R \sim 1$, and we proceed under this assumption. The total p time separating our present epoch from the end of the radiation era is computed from the equations above [and $k = 0$], obtaining

$$p_0 \simeq \ln\left[3\widetilde{H}^2/(8\pi\widetilde{G}\tilde{\rho}_{\rm radiation})\right]_0 + a(\varphi_R) - a(\varphi_0) \simeq 10$$

(using $\widetilde{H} \simeq 75$ km s^{-1}Mpc^{-1} and $\kappa \sim 1$). We now have the ingredients needed in Eq. (6) to give a numerical estimate of the deviation from general relativity to be expected at the present epoch: in terms of the post-Newtonian parameter γ we obtain (with $\theta_0 \equiv \omega p_0 + \theta_R$)

$$1 - \gamma = \frac{2\alpha_0^2}{1 + \alpha_0^2} \simeq \frac{2\alpha_R^2}{1 - 3/(8\kappa)} \sin^2\theta_0\, e^{-\frac{3}{2}p_0} \sim 3 \times 10^{-7} \ , \tag{7}$$

when considering values of κ sufficiently above 3/8, and using $\alpha_R \sim 1$ and $\sin^2\theta_0 \sim 1/2$.

This "strong" curvature ($\kappa > 3/8$) case represents the lower bound on the predicted magnitude of $1-\gamma$. Indeed, similar estimates for the other attractor scenarios (local minimum with $\kappa \leq 3/8$, minimum at infinity in field space $[a(\varphi) \propto \varphi^{-n}]$) generically yield substantially higher values for $1-\gamma$. For instance, the attraction in a parabolic potential of "weak" curvature ($\kappa < 3/8$) exhibits a purely damped relaxation toward the minimum with an efficiency such that

$$1-\gamma \simeq 2\alpha_R^2 e^{-2\kappa p_0} \ , \tag{8}$$

when κ is sufficiently below 3/8. The efficiency (8) is always less than the one in the critically damped case ($\kappa = 3/8$), in which

$$1-\gamma \simeq 2\alpha_R^2 \left(1+\frac{3}{4}p_0\right)^2 e^{-\frac{3}{2}p_0} \sim 4\times 10^{-5} \ . \tag{9}$$

This analysis has been generalized to the case of negative spatial curvature cosmologies [9]. Again we find that the analog of Eq. (7) is a lower bound, and that (under the assumption $\alpha_R \sim 1$) the theoretically likely level of deviation from general relativity is sensitively dependent on the value of the present total cosmological matter density,

$$1-\gamma \sim 2\alpha_R^2 \left(\frac{\tilde{\rho}_0^{\text{matter}}}{10^{-30}\text{g cm}^{-3}}\right)^{-3/2} \times 10^{-5} \tag{10}$$

[where $\tilde{\rho}_0^{\text{matter}} = 10^{-30}\text{g cm}^{-3}$ corresponds to $\widetilde{\Omega} = \tilde{\rho}_0^{\text{matter}}/\tilde{\rho}^{\text{critical}} \simeq 0.1$ if $\widetilde{H} = 75\,\text{km s}^{-1}\text{Mpc}^{-1}$].

Correlatively to the above predictions for $1-\gamma$, the post-Newtonian parameter β is given by

$$\beta - 1 = \frac{1}{8}\kappa(1+\gamma)(1-\gamma) \ ,$$

and the present time variation of the effective Newtonian coupling "constant" $\widetilde{G} = G_* A^2(1+\alpha^2)$ is at the level

$$\left|(\dot{\widetilde{G}}/\widetilde{G})_0\right| \sim (1+\kappa)(1-\gamma)\widetilde{H}_0 \simeq (4\beta-\gamma-3)\widetilde{H}_0 \ .$$

This latter connection indicates values for $\dot{\widetilde{G}}$ which will be difficult to detect in solar-system experiments.

In conclusion, tensor-scalar theories of gravity generically contain a natural attractor mechanism tending to drive the world toward a state close to a pure general relativistic one. The large but finite redshift factor separating us from the end of the radiation-dominated era provides the measure for the expected present deviations from general relativity, and leads us to estimate that they are small but not unmeasurably small. The numerical estimates, Eqs. (7)–(10), should hopefully provide new, strong motivations for experiments which push beyond the present empirical upper bounds on the post-Newtonian parameters $\gamma-1$ or $\beta-1$.

The scientific implications of non zero results for them would be enormous: it would signal the existence of a new long-range interaction.

The work of K.N. was supported by the French Académie des Sciences (Maxwell Foundation).

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[13] Note however that the evolution during the inflation era ($\lambda = -1$) also exhibits a generic tendency to be trapped near a minimum of $a(\varphi)$. This may mean that inflation requires special shapes of $a(\varphi)$ and special initial conditions to terminate with $\alpha > \alpha_{\rm min}$.