$$\begin{tikzcd} X \arrow[r, hook, "i"] \arrow[dr, "f"', "n"] & Y \arrow[d, "g", "N\text{ lisse}"'] \\ & S \end{tikzcd}$$

régulière

~~n lisse~~, $f^!\mathcal{O} = \omega_n[n]$

~~$\Omega^n_X$~~

$f^!\mathcal{O} = i^! g^! \mathcal{O} = R\mathcal{H}om_{\mathcal{O}_Y}(\mathcal{O}_X, \Omega^N_Y)[N]$

$\omega_n = R\mathcal{H}om_{\mathcal{O}_Y}(\mathcal{O}_X, \Omega^N_Y)[N-n]$

$= \mathcal{H}om(\bigwedge^d I/I^2, \Omega^N_Y \otimes \mathcal{O}_X)[-d][N-n]$

$= \underline{\mathcal{H}om}(\bigwedge^d I/I^2, \Omega^N_Y \otimes \mathcal{O}_X) = \bigwedge^d (I/I^2)^\vee \otimes \Omega^N_Y$ = dét $\Omega^n_{X/S}$

~~$\mathcal{H}om(\Omega^n_X, \omega_n)$~~ si $X$ lisse, dét $\Omega^1_X \cong \Omega^n_X$.

~~$\Omega^n_X$~~

$$\begin{tikzcd} \bigwedge^d(I/I^2) \arrow[r] & \Omega^N_Y \otimes \mathcal{O}_X \arrow[r] \arrow[d, dotted] & \Omega^n_X \arrow[r] & 0 \\ & \bigwedge^d(I/I^2)^\vee \otimes \Omega^N_Y \otimes \mathcal{O}_X & & \end{tikzcd}$$

$\mathcal{H}om(\Omega^n_Y \otimes \mathcal{O}_X, \bigwedge^d N^\vee \otimes \Omega^N_Y) = H^0(X, \bigwedge^d N^\vee \otimes \Omega^\lambda_Y)$.